Analysis of Total, Direct and Indirect Causal Effects in Experiments and Quasi-Experiments

Prof. Dr. Rolf Steyer

University of Jena
Institute of Psychology
Chair of Methodology and Evaluation Research

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- Theory of causal effects
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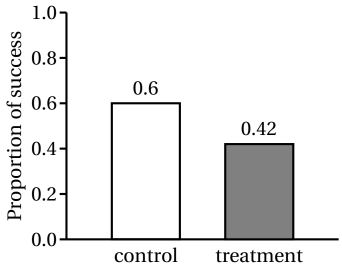
Why we need a theory of causality

Simpson's Paradox

Total sample

Success	Treatment no (X=0)	 yes (X=1)	
no (Y=0)	240	232	472
yes (Y=1)	360	168	528
	600	400	1000

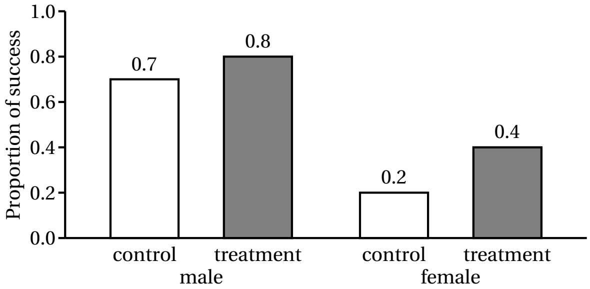
360/600 = .60 168/400 = .42



Simpson's Paradox: Males vs. Females

Success	Males (Z=0) Control (X=0)	Males (Z=0) Treatment (X=1)	Females (Z=1) Control (X=0)	Females (Z=1) Treatment (X=1)
No (Y=0)	144	4	96	228
Yes (Y=1)	336	16	24	152
	480	20	120	380

336/480 = .70   16/20 = .80   24/120 = .20   152/380 = .40



Nonorthogonal ANOVA I

Table 1: Expectations in three treatment conditions

treatment	expectation of Y in the treatment conditions $E(Y \mid X{=}x)$	treatment probabilities $P(X{=}x)$
X=0 (control)	111.25	1/3
X=1 (treatment 1)	100.00	1/3
X=2 (treatment 2)	114.25	1/3
$E(Y)$	108.50	

Nonorthogonal ANOVA II

Table 2: Expectations $E(Y \mid X{=}x, Z{=}z)$ in treatment × neediness conditions

	neediness						
treatment	low (Z=0)		medium (Z=1)		high (Z=2)		$P(X{=}x)$
X=0	120	(20/120)	110	(17/120)	60	(3/120)	(40/120)
X=1	100	(7/120)	100	(26/120)	100	(7/120)	(40/120)
X=2	80	(3/120)	90	(17/120)	140	(20/120)	(40/120)
$P(Z{=}z)$		(30/120)		(60/120)		(30/120)	

Note. Probabilities $P(X{=}x, Z{=}z)$, $P(Z{=}z)$, and $P(X{=}x)$ in parentheses.

Direct Treatment Effect: Path Diagram I

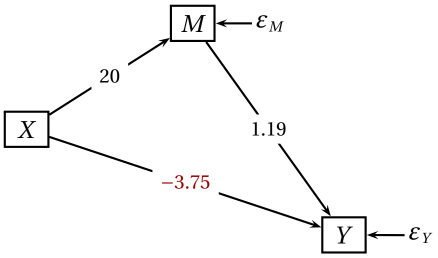


Figure 1: Path diagram with a randomized treatment variable X, a mediator M (post-test motivation), and an outcome variable Y (post-test achievement).

$$E(Y \mid X) \;=\; 130 + 20 \cdot X$$

$$E(M \mid X) \;=\; 80 + 20 \cdot X$$

$$E(Y \mid X, M) \;\approx\; 34.9924 - 3.7528 \cdot X + 1.1876 \cdot M$$

Direct Treatment Effect: Expectations, Covariances, and Correlations

Table 3: Covariances, Correlations, and Expectations in the Teaching Experiment

		W	Z	X	M	Y
Pre-test achievement	W	100.00	*.850*	*.000*	*.495*	*.740*
Pre-test motivation	Z	85.00	100.00	*.000*	*.582*	*.696*
Treatment (yes/no)	X	0.00	0.00	0.25	*.727*	*.597*
Post-test motivation	M	68.00	80.00	5.00	189.00	*.893*
Post-test achievement	Y	124.00	116.50	5.00	205.70	280.45
Expectations		100.00	100.00	0.50	90.00	140.00

Note. Correlations (in italics) are rounded.

Direct Treatment Effect: Path Diagram II

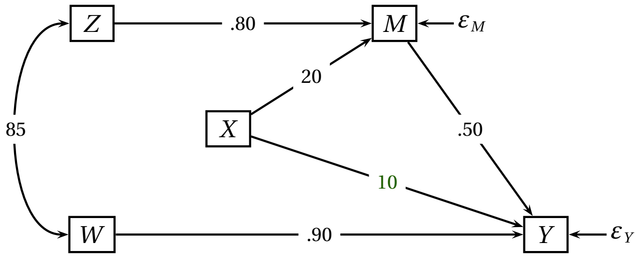


Figure 2: Path diagram with a randomized treatment variable X, two pre-tests Z (pre-test motivation) and W (pre-test achievement), a mediator M (post-test motivation), and an outcome variable Y (post-test achievement).

$$E(Y \mid X) \;=\; 130 + 20 \cdot X$$

$$E(Y \mid X, M) \;\approx\; 34.9924 - 3.7528 \cdot X + 1.1876 \cdot M$$

$$E(Y \mid X, Z, M) \;=\; 13.50 + 10 \cdot X + .50 \cdot M + .765 \cdot Z$$

$$E(Y \mid X, Z, M, W) \;=\; 0 + 10 \cdot X + 0 \cdot Z + .50 \cdot M + .90 \cdot W$$

Generalized ANCOVA

Traditional ANCOVA Model

If Z is a univariate covariate, the treatment variable X takes values $0, 1, \dots, J$, and the random variables $I_{X=x}$ indicate with their values 1 and 0 whether or not $X{=}x$, *traditional analysis of covariance* assumes

$$E(Y \mid X, Z) \;=\; \gamma_{00} + \gamma_{01} \cdot Z + \sum_{x=1}^{J} \gamma_{x0} \cdot I_{X=x}. \tag{1}$$

For $X{=}0$, this equation yields:

$$E_{X=0}(Y \mid Z) \;=\; \gamma_{00} + \gamma_{01} \cdot Z, \tag{2}$$

and for $X{=}x$, Equation (1) yields:

$$E_{X=x}(Y \mid Z) \;=\; \gamma_{00} + \gamma_{01} \cdot Z + \gamma_{x0}\,. \tag{3}$$

Generalized ANCOVA Model I

The fundamental equation for generalized analysis of covariance is:

$$E(Y \mid X, Z) \;=\; g_0(Z) + \sum_{x=1}^{J} g_x(Z) \cdot I_{X=x}, \tag{4}$$

where the *intercept function* $g_0(Z)$ and the *effect functions* $g_x(Z)$ are unknown functions of the (possibly multivariate, numerical or non-numerical) covariate Z.

Remember, *traditional analysis of covariance* assumes

$$E(Y \mid X, Z) \;=\; \gamma_{00} + \gamma_{01} \cdot Z + \sum_{x=1}^{J} \gamma_{x0} \cdot I_{X=x}. \tag{5}$$

Generalized ANCOVA Model II

The fundamental equation for generalized analysis of covariance is:

$$E(Y \mid X, Z) \;=\; g_0(Z) + \sum_{x=1}^{J} g_x(Z) \cdot I_{X=x}, \tag{6}$$

where the *intercept function* $g_0(Z)$ and the *effect functions* $g_x(Z)$ are unknown functions of the (possibly multivariate, numerical or non-numerical) covariate Z.

If X is discrete this equation is *always* true as long as no restrictive assumptions about the intercept and/or effect functions are introduced.

Conditioning on the covariate, Equation (6) yields

$$E_{Z=z}(Y \mid X) \;=\; g_0(z) + \sum_{x=1}^{J} g_x(z) \cdot I_{X=x}. \tag{7}$$

This equation shows that the effects of the treatments may be different for different values of the covariate.

Generalized ANCOVA Model III

Conditioning on the treatment, Equation (6) yields, for $X{=}0$

$$E_{X=0}(Y \mid Z) \;=\; g_0(Z), \tag{8}$$

and for $X{=}x$:

$$E_{X=x}(Y \mid Z) \;=\; g_0(Z) + g_x(Z). \tag{9}$$

Hence,

$$g_x(Z) \tag{10}$$

is the (prima facie) effect function, comparing treatment x to treatment 0 and

$$E[g_x(Z)] \tag{11}$$

is the *average effect* of treatment x compared to treatment 0.

If $x = 0, \dots, J$ denote the values of X, in generalized ANCOVA, we estimate both the conditional-effect functions $g_x(Z)$ and the average effects $E[g_x(Z)]$, for $x = 1, \dots, J$.

EffectLite

Scope of *EffectLite* I

EffectLite ...

- does not assume homogeneity of variances (in the univariate case with a single outcome variable) or of covariance matrices (in the multivariate case with two or more outcome variables) of the outcome variables between treatment groups.
- allows analyzing mean differences between groups w. r. t. several manifest outcome variables, one or more *latent outcome variables*, and a mixture of the two kinds of outcome variables.
- allows analyzing conditional and average effects w. r. t. several manifest covariates or w. r. t. one or more *latent covariates*, and a mixture of the two kinds of covariates.
- estimates and tests average effects for non-orthogonal analysis of variance designs, provided that the covariates are specified as qualitative indicator variables. Other programs typically do not test the average effect at all,or they do not treat the covariates as stochastic regressors, which usually leads to invalid tests of the average effect.
- produces results which are easily interpretable in the analysis of conditional and of average effects (mean differences) between groups.

Scope of *EffectLite* II

EffectLite ...

- estimates and tests *conditional* and *average total effects*, provided that the covariate-treatment regression is unbiased.
- estimates and tests *conditional* and *average direct effects*, provided that the covariate-mediator-treatment regression is unbiased.
- needs Mplus or LISREL in the background
- works with demo version or student version for small models

Theory of Causal Effects 19 / 45

Probability and Causality

Probability and Causality

Rolf Steyer
Ivailo Partchev
Ulf Kröhne
Benjamin Nagengast
Christiane Fiege

Springer Texts in Statistics. New York: Springer, 2011.

draft of many chapters available at www.causal-effects.de

also contains the *Causal Effects Explorer*

and *EffectLite* (with manual).

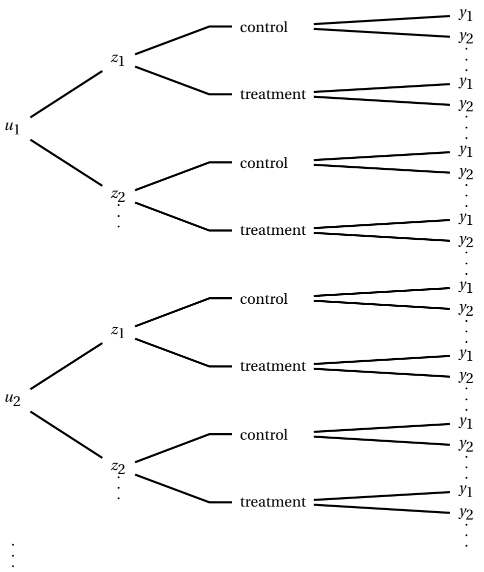
Single-Unit Trials

To which type of *empirical phenomenon* does the theory refer?

- Drawing an observational unit u (e.g., a person) out of a set of units,
- observing the value z of a (possibly multivariate qualitative or quantitative) Z of the unit, if such a covariate is of interest,
- assigning the unit or observing its assignment to one of several experimental conditions (represented by the value x of the treatment variable X),
- recording the numerical value y of the outcome variable Y.

In this single-unit trial, the units u can be considered to be the values of a non-numerical random variable U, and all four random variables, U, Z, X and Y have a joint distribution.

Tree Diagram of a Single-Unit Trial



Introduction: The Problem

- Outside the randomized experiment, i.e., in quasi-experiments, the conditional expectations $E(Y \mid X{=}x)$ of the outcome variable Y given treatment $X{=}x$ and their differences between treatment conditions usually are biased
- The difference $E(Y \mid X{=}x) - E(Y \mid X{=}x')$ can have a different sign than the causal effect (due to selection bias, non-comparable treatment and control groups, etc.)
- If it is not the conditional expectations $E(Y \mid X{=}x)$ and their differences we should estimate and test, what is it that we are after in experimental and quasi-experimental research?
- General answer: Causal effects. But what are causal effects?

Introduction: Rubin's Answer

- Rubin's *individual causal effects*

$$\textit{Rubin's ICE}_{xx'} \;\equiv\; Y_x(u) - Y_{x'}(u) \tag{12}$$

$Y_x(u)$ denotes the potential outcome of unit u under treatment x and $Y_{x'}(u)$ its potential outcome under treatment x'. $Y_x(u)$ and $Y_{x'}(u)$ are fixed numbers.

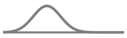
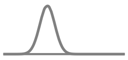
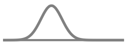
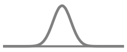
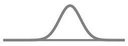
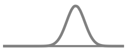
- Rubin's *average causal effect*

$$\textit{Rubin's ACE}_{xx'} \;\equiv\; \frac{1}{N}\sum_{u=1}^{N}\left[Y_x(u) - Y_{x'}(u)\right] \tag{13}$$

Shortcomings of these concepts

- This deterministic outcome assumption, assuming $Y_x(u)$ and $Y_{x'}(u)$ to be fixed numbers, ignores the problem of multiple determinacy
- There is even bias at the individual unit level, and this bias is ignored.

Table illustrating Neyman's true yields

Observational-unit variable U (plots)	Covariate 'soil fertility' Z	True yield of variety A $E(Y \mid X{=}0, U{=}u)$	Intra-individual distribution of the outcome variable under control	True yield of variety B $E(Y \mid X{=}1, U{=}u)$	Intra-individual distribution of the outcome variable under treatment	Individual causal effect $E(Y \mid X{=}1, U{=}u)$ – $E(Y \mid X{=}0, U{=}u)$
u_1	low	68		82		14
u_2	low	81		89		8
u_3	low	89		101		12
u_4	low	92		108		16
u_5	high	112		118		6
u_6	high	119		123		4

Introduction: Neyman's Answer

- Neyman's *individual causal effects*

$$\textit{Neyman's ICE}_{xx'} \equiv E(Y \mid X{=}x, U{=}u) - E(Y \mid X{=}x', U{=}u)$$

Neyman calls the expectations $E(Y \mid X{=}x, U{=}u)$ "true yields".

- Neyman's *average causal effect*

$$\textit{Neyman's ACE}_{xx'} \equiv \frac{1}{N}\sum_{u=1}^{N}\big[E(Y \mid X{=}x, U{=}u) - E(Y \mid X{=}x', U{=}u)\big]$$

- No deterministic outcome assumption, i.e., it is an appropriate answer to the problem of multiple determinacy
- It is not a good answer to the problem of bias at the individual unit level

Stratification or Conditioning

- Both, Neyman and Rubin, base their concepts on stratifying or conditioning on the individual level
- Atomic stratification means stratifying or *conditioning on all potential confounders*, even those that are beyond the individual level
- Hence, we simply replace Neyman's conditional expectation $E_{X=x}(Y \mid U)$ by $E_{X=x}(Y \mid D_X)$, where D_X is the *comprehensive potential confounder.*

Filtration $(\mathfrak{C}_t)_{t \in T}$ and Confounder σ-algebra $\mathfrak{D}_X$

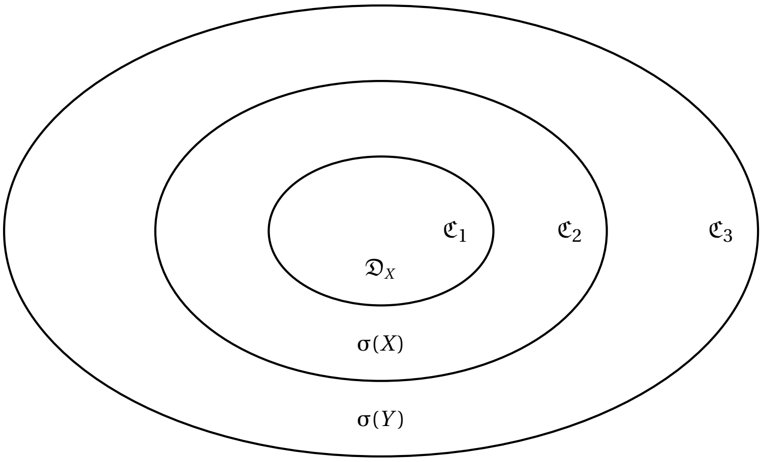


Figure 3: Venn-diagram of a filtration $(\mathfrak{C}_t)_{t \in T}$ with $T = \{1, 2, 3\}$. $\sigma(X)$ denotes the σ-algebra generated by the random variable X.

Probability Space and Causality Space

A *probability space* $(\Omega, \mathfrak{A}, P)$ consists of

- a set Ω of possible outcomes (of the random experiment)
- a σ-algebra $\mathfrak{A}$ of possible events
- a probability measure $P : \mathfrak{A} \to [0, 1]$

A *causality space* $\langle (\Omega, \mathfrak{A}, P), (\mathfrak{C}_t)_{t \in T}, X, Y, \mathfrak{D}_X \rangle$ consists of:

- a probability space $(\Omega, \mathfrak{A}, P)$
- a random variable X (putative cause)
- a random variable Y (outcome variable)
- a filtration $(\mathfrak{C}_t)_{t \in T}$ (w. r. t. which random variables and events can be ordered)
- a potential confounder σ-algebra $\mathfrak{D}_X$ (w. r. t. which potential confounders are measurable)

True Outcome Variables and True Effect Variables

- Using the comprehensive potential confounder D_X and the conditional expectation $E_{X=x}(Y \mid D_X)$ of Y given D_X in treatment x, we can define the *true outcome variables* $\tau_x \equiv E_{X=x}(Y \mid D_X)$ and the *true effect total variables* $\delta_{xx'} \equiv \tau_x - \tau_{x'}$
- These variables τ_x and $\delta_{xx'}$ are, by definition, purged from confounding
- τ_x and $\delta_{xx'}$ are random variables on the same probability space as the original random variables X and Y. They have expectations, conditional expectations, variances, covariances, etc.
- The true outcome variables τ_x play the same role as Rubin's potential outcome variables Y_x.

True Outcomes: Example

Table 4: Joe and Ann Self-Selected, With (X=x)-Conditional Regressions

Outcomes ω			Observables			Regressions					
Unit	Treatment	Success	Observational-unit variable U	Treatment variable X	Outcome variable Y	Regression $E(Y \mid X, U)$	Regression $E(Y \mid X)$	Conditional probability $P(X{=}1 \mid U) = E(I_{X=1} \mid U)$	Conditional regression $E_{X=0}(Y \mid U)$	Conditional regression $E_{X=1}(Y \mid U)$	Probabilities of elementary events $P(\{\omega\})$
(*Joe*,	*no*,	−)	*Joe*	0	0	.70	.60	.04	.70	.80	.144
(*Joe*,	*no*,	+)	*Joe*	0	1	.70	.60	.04	.70	.80	.336
(*Joe*,	*yes*,	−)	*Joe*	1	0	.80	.42	.04	.70	.80	.004
(*Joe*,	*yes*,	+)	*Joe*	1	1	.80	.42	.04	.70	.80	.016
(*Ann*,	*no*,	−)	*Ann*	0	0	.20	.60	.76	.20	.40	.096
(*Ann*,	*no*,	+)	*Ann*	0	1	.20	.60	.76	.20	.40	.024
(*Ann*,	*yes*,	−)	*Ann*	1	0	.40	.42	.76	.20	.40	.228
(*Ann*,	*yes*,	+)	*Ann*	1	1	.40	.42	.76	.20	.40	.152

Example Illustrating the Basic Concepts

Fundamental parameters						Derived parameters				
Unit u	Sampling probability $P(U{=}u)$	Covariate gender Z	True outcome under control $\tau_0(u) = E(Y \mid X{=}0, U{=}u)$	True outcome under treatment $\tau_1(u) = E(Y \mid X{=}1, U{=}u)$	Individual treatment probability $P(X{=}1 \mid U{=}u)$	Individual total effect $\delta_{10} = \tau_1(u) - \tau_0(u)$	$P(U{=}u \mid X{=}0)$	$P(U{=}u \mid X{=}1)$	Regression $E_{X=0}(Y \mid Z)$	Regression $E_{X=1}(Y \mid Z)$
u_1	1/6	m	68	81	3/4	13	1/10	3/14	83	92.5
u_2	1/6	m	78	86	3/4	8	1/10	3/14	83	92.5
u_3	1/6	m	88	100	3/4	12	1/10	3/14	83	92.5
u_4	1/6	m	98	103	3/4	5	1/10	3/14	83	92.5
u_5	1/6	f	106	114	1/4	8	3/10	1/14	111	122
u_6	1/6	f	116	130	1/4	14	3/10	1/14	111	122
	Expectations		92.33	102.33	7/12	10			92.33	102.33
	$E(Y \mid X{=}x)$		99.80	96.71						

Average total effect (*ACE*)	10.000
Prima facie effect (*PFE*)	−3.086

	male	female
Conditional total effects	9.50	11.00
Conditional *PFE*s	9.50	11.00

Basic Concepts of the Theory of Causal Effects

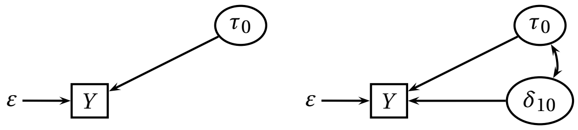
Primitives

$\Omega = \Omega_U \times \Omega_Z \times \Omega_X \times \Omega_Y$	The *set of possible outcomes*
$U : \Omega \rightarrow \Omega_U$	*Observational-unit variable*
$Z_i : \Omega \rightarrow \Omega'_{Z_i}$	*Covariates*
$X : \Omega \rightarrow \{0, 1, \ldots, J\}$	*Treatment variable*
$Y : \Omega \rightarrow \mathbb{R}$	*Outcome variable*

Theoretical Concepts the Theory of Causal Effects

$\tau_x \equiv E_{X=x}(Y \mid D_X)$	*True outcome variables in treatment conditions x with respect to total effects*
$\delta_{xx'} \equiv \tau_x - \tau_{x'}$	*True total effect variables*
$E(\delta_{xx'})$	*Average total effect*
$E(\delta_{xx'} \mid Z{=}z)$	*Conditional total effect given Z=z*
$E(\delta_{xx'} \mid X{=}x^*)$	*Conditional total effect given X=x**

Path Diagram



(a) control (b) treatment

$$Y = \tau_0 + \varepsilon \qquad\qquad Y = \tau_0 + (\tau_1 - \tau_0) + \varepsilon = \tau_0 + \delta_{10} + \varepsilon$$

Decomposition of variances
... in control group

$$Var(Y \mid X{=}0) = Var(\tau_0 \mid X{=}0) + Var(\varepsilon \mid X{=}0) \tag{14}$$

... in treatment group

$$\begin{aligned} Var(Y \mid X{=}1) = {} & Var(\tau_0 \mid X{=}1) + Var(\varepsilon \mid X{=}1) \\ & + Var(\delta_{10} \mid X{=}1) + 2 \cdot Cov(\tau_0, \delta_{10} \mid X{=}1) \end{aligned} \tag{15}$$

Identification of Total Effects

Core of the Theory of Total Effects

Notation

X — treatment variable Z — covariate
Y — outcome variable U — observational-unit variable

$\tau_x \equiv E_{X=x}(Y \mid D_X)$ — true outcome variable
$\delta_{xx'} \equiv \tau_x - \tau_{x'}$ — true total effect variable
$E(\delta_{xx'})$ — average total effect
$E(\delta_{xx'} \mid Z{=}z)$ — conditional total effect given $Z{=}z$

Unbiasedness

…of the expectation $E(Y \mid X{=}x)$ $:\Leftrightarrow$ $E(Y \mid X{=}x) = E(\tau_x)$
…of the regression $E_{X=x}(Y \mid Z)$ $:\Leftrightarrow$ $E_{X=x}(Y \mid Z) \underset{a.s.}{=} E(\tau_x \mid Z)$

Unbiasedness of the regression $E_{X=x}(Y \mid Z)$ implies:

$$(1) \quad E\big[E_{X=x}(Y \mid Z)\big] = E\big[E(\tau_x \mid Z)\big] = E(\tau_x) \qquad E(\delta_{xx'}) = E(\tau_x) - E(\tau_{x'})$$

and, if $V = f(Z)$,

$$(2) \quad E\big[E_{X=x}(Y \mid Z) \mid V\big] \underset{a.s.}{=} E\big[E(\tau_x \mid Z \mid V)\big] \underset{a.s.}{=} E(\tau_x \mid V)$$

$$E(\delta_{xx'} \mid V) \underset{a.s.}{=} E(\tau_x \mid V) - E(\tau_{x'} \mid V)$$

Each Condition is Sufficient for Unbiasedness

- Z-conditional independence of potential confounders and treatments ($X \perp\!\!\!\perp D_X \mid Z$)

$$P(X{=}x \mid D_X, Z) \underset{a.s.}{=} P(X{=}x \mid Z) \qquad \text{for each value } x \text{ of } X$$

- Completeness of the regression ($Y \vdash D_X \mid X, Z$)

$$E(Y \mid X, Z, D_X) \underset{a.s.}{=} E(Y \mid X, Z)$$

- Z-Conditional Strong Causality

$$E(Y \mid X, Z, D_X) \underset{a.s.}{=} E(Y \mid X, Z) + W, \quad \text{where } W \text{ is } D_X\text{-measurable}$$

- Z-conditional unconfoundedness of the regression $E(Y \mid X, Z)$

$$E(Y \mid X{=}x, Z{=}z) = \int E(Y \mid X{=}x, Z{=}z, W{=}w)\, P^{W \mid Z=z}(dw)$$

for $P^{X,Z}$-almost all $(x, z) \in \Omega'_X \times \Omega'_Z$
and all potential confounders W.

Each Condition is Sufficient for Unbiasedness

- Z-conditional independence of true outcomes and treatments
  ($\tau \perp\!\!\!\perp X \mid Z$ "strong ignorability")

$$P(X{=}x \mid Z, \tau_0, \tau_1, \ldots, \tau_J) \underset{a.s.}{=} P(X{=}x \mid Z) \qquad \text{for each value } x \text{ of } X$$

- Z-conditional regressive independence of true outcomes from treatments ($\tau \vdash X \mid Z$)

$$E(\tau_x \mid X, Z) \underset{a.s.}{=} E(\tau_x \mid Z) \qquad \text{for each value } x \text{ of } X$$

Unfortunately, both conditions are not empirically testable.

Implication Structure Among Conditions of Unbiasedness

Table 5: Implication Structure Between Causality Conditions

		(iii)	(iv)	(vi)	(vii)
(i)	Z-conditional independence of X and potential confounders $(X \perp\!\!\!\perp D_X \mid Z)$	⇒	⇒	⇒	⇒
(ii)	Completeness of $E(Y \mid X, Z)$ $(Y \vdash D_X \mid X, Z)$	⇒	⇒	⇒	⇒
(iii)	Z-conditional independence of X and true outcomes $(X \perp\!\!\!\perp \tau \mid Z)$		⇒		⇒
(iv)	Z-conditional regressive independence of true outcomes from X $(\tau \vdash X \mid Z)$				⇒
(v)	Strong causality of $E(Y \mid X, Z)$ w. r. t. Z-conditional total effects			⇒	⇒
(vi)	Unconfoundedness of $E(Y \mid X, Z)$ w. r. t. Z-conditional total effects				⇒
(vii)	Unbiasedness of $E(Y \mid X, Z)$ w. r. t. Z-conditional total effects				⇒

Note: ⇒ indicates that condition in row implies condition in column.

Identification of Direct Effects

Core of the Theory of Direct Effects

Notation

X — treatment variable Z — covariate
Y — outcome variable U — observational-unit variable
M — mediator V — any function of Z and M

$\tau_{x;\,M} \equiv E_{X=x}(Y \mid D_X, M)$ — true outcome variable w. r. t. direct effects
$\delta_{xx';\,M} \equiv \tau_{x;\,M} - \tau_{x';\,M}$ — true direct effect variable
$E(\delta_{xx';\,M})$ — average direct effect
$E(\delta_{xx';\,M} \mid V{=}v)$ — conditional direct effect given $V{=}v$

Unbiasedness of the regression $E_{X=x}(Y \mid Z, M)$

$$E_{X=x}(Y \mid Z, M) \underset{a.s.}{=} E(\tau_{x;\,M} \mid Z, M)$$

Unbiasedness of the regression $E_{X=x}(Y \mid Z, M)$ implies:

$$(1) \quad E\big[E_{X=x}(Y \mid Z, M)\big] = E\big[E(\tau_{x;\,M} \mid Z, M)\big] = E(\tau_{x;\,M})$$

Remember : $E(\delta_{xx';\,M}) = E(\tau_{x;\,M}) - E(\tau_{x';\,M})$

and

$$(2) \quad E\big[E_{X=x}(Y \mid Z, M) \mid V\big] \underset{a.s.}{=} E\big[E(\tau_{x;\,M} \mid Z, M) \mid V\big] = E(\tau_{x;\,M} \mid V)$$

Remember : $E(\delta_{xx';\,M} \mid V) \underset{a.s.}{=} E(\tau_{x;\,M} \mid V) - E(\tau_{x';\,M} \mid V)$

Each Condition is Sufficient for Unbiasedness w. r. t. Direct Effects

- (Z, M)-conditional independence of potential confounders and treatments $(X \perp\!\!\!\perp D_X \,|\, Z, M)$

$$P(X{=}x \,|\, D_X, Z, M) \underset{a.s.}{=} P(X{=}x \,|\, Z, M) \qquad \text{for each value } x \text{ of } X$$

- Completeness of the regression $(Y \vdash D_X \,|\, X, Z, M)$

$$E(Y \,|\, X, Z, M, D_X) \underset{a.s.}{=} E(Y \,|\, X, Z, M)$$

- Strong Causality

$$E(Y \,|\, X, Z, M, D_X) \underset{a.s.}{=} E(Y \,|\, X, Z, M) + W, \quad \text{where } W \text{ is } D_X\text{-measurable}$$

- Unconfoundedness of the regression $E(Y \,|\, X, Z, M)$ w. r. t. (Z, M)-conditional M-direct effects

$$E(Y \,|\, X{=}x, Z{=}z, M{=}m) = \int E(Y \,|\, X{=}x, Z{=}z, M{=}m, W{=}w)\, P^{W \,|\, Z=z, M=m}(dw)$$

$$\text{for } P^{X,Z,M}\text{-almost all } (x, z, m) \in \Omega'_X \times \Omega'_Z \times \Omega'_M$$

and all potential confounders W.

Each Condition is Sufficient for Unbiasedness w. r. t. Direct Effects

- (Z, M)-conditional independence of true outcomes and treatments
  $(\tau_M \perp\!\!\!\perp X \,|\, Z, M$ "strong ignorability")

$$P(X{=}x \,|\, Z, M, \tau_{0;M}, \tau_{1;M}, \ldots, \tau_{J;M}) \underset{a.s.}{=} P(X{=}x \,|\, Z, M) \qquad \text{for each value } x \text{ of } X$$

- (Z, M)-conditional regressive independence of true outcomes from treatments $(\tau_M \vdash X \,|\, Z, M)$

$$E(\tau_{x;\,M} \,|\, X, Z, M) \underset{a.s.}{=} E(\tau_{x;\,M} \,|\, Z, M) \qquad \text{for each value } x \text{ of } X$$

Both conditions are not empirically testable.

Implication Structure Among Conditions of Unbiasedness w. r. t. Direct Effects

Table 6: Implication Structure Between Causality Conditions

		(iii)	(iv)	(vi)	(vii)
(i)	(Z, M)-conditional independence of X and potential confounders $(X \perp\!\!\!\perp D_X \,|\, Z, M)$	⇒	⇒	⇒	⇒
(ii)	Completeness of $E(Y \,|\, X, Z, M)$ $(Y \vdash D_X \,|\, X, Z, M)$	⇒	⇒	⇒	⇒
(iii)	(Z, M)-conditional independence of X and true outcomes $(X \perp\!\!\!\perp \tau_M \,|\, Z, M)$		⇒		⇒
(iv)	(Z, M)-conditional regressive independence of true outcomes from X $(\tau_M \vdash X \,|\, Z, M)$				⇒
(v)	Strong causality of $E(Y \,|\, X, Z, M)$ w. r. t. (Z, M)-conditional M-direct effects			⇒	⇒
(vi)	Unconfoundedness of $E(Y \,|\, X, Z, M)$ w. r. t. (Z, M)-conditional M-direct effects				⇒
(vii)	Unbiasedness of $E(Y \,|\, X, Z, M)$ w. r. t. (Z, M)-conditional M-direct effects				⇒

Note: ⇒ indicates that condition in row implies condition in column.

Conclusion

Outside the randomized experiment, traditional statistical methods including regression, analysis of variance, and structural equation modeling, are not sufficient to estimate and test intervention effects. Within the perfect randomized experiment, these skills only suffice to estimate and test the average total effect as well as the conditional total effects given covariates such as pre-tests, gender, or educational status. The analysis of direct and indirect effects – even within the randomized experiment, just like the analysis of causal effects in quasi-experiments – inevitably involves the theory of causal effects and its implications for the design of studies and techniques of data analysis that aim at the analysis of causal effects.