Latent-State-Trait-Theorie

Grundbegriffe der Latent-State-Trait-Theorie

Die Menge der möglichen

Ergebnisse des Zufallsexperiments

$$\Omega = \Omega_U \times \Omega_{S_1} \times \ldots \times \Omega_{S_k} \times \ldots \times \Omega_{S_n}$$

$$\times\, \Omega_{O_1} \times \ldots \times \Omega_{O_k} \times \ldots \times \Omega_{O_n}$$

Testwertvariablen

$Y_{ik}\colon \Omega \rightarrow \mathbb{R}$

Projektionen

$U\colon \Omega \rightarrow \Omega_U$ Personprojektion

$S_k\colon \Omega \rightarrow \Omega_{S_k}$ Situationsprojektionen

Latent-State-Trait-Theorie

Latente Variablen

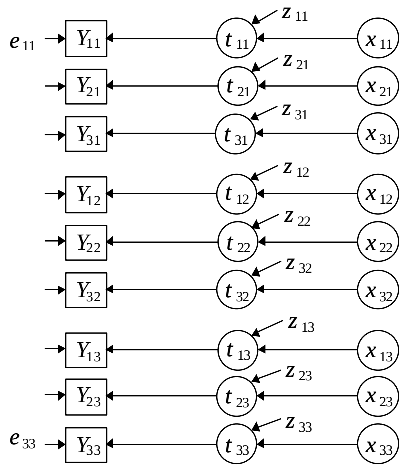
$t_{ik} := E(Y_{ik} \mid U, S_k)$ Latent-Statevariable

$e_{ik} := Y_{ik} - t_{ik}$ Meßfehlervariable

$x_{ik} := E(Y_{ik} \mid U)$ Latent-Traitvariable

$z_{ik} := t_{ik} - x_{ik}$ Latent-Stateresiduum

Latent-State-Trait-Theorie



Latent-State-Trait-Theorie

Eigenschaften der Latenten Variablen

Dekomposition der Variablen

$$Y_{ik} = t_{ik} + e_{ik}$$

$$t_{ik} = x_{ik} + z_{ik}$$

Dekomposition der Varianzen

$$Var(Y_{ik}) = Var(t_{ik}) + Var(e_{ik})$$

$$Var(t_{ik}) = Var(x_{ik}) + Var(z_{ik})$$

Latent-State-Trait-Theorie

Erwartungswerte und Kovarianzen der Residuen

$$E(e_{ik}) = 0$$

$$E(z_{ik}) = 0$$

$$Cov(e_{ik}, z_{jl}) = 0$$

$$Cov(e_{ik}, t_{jl}) = 0$$

$$Cov(e_{ik}, x_{jl}) = 0$$

$$Cov(z_{ik}, x_{jl}) = 0$$

Latent-State-Trait-Theorie

Wichtige Kenngrößen der Latent-State-Trait-Theorie

Reliabilität

$$Rel(Y_{ik}) = \frac{Var(t_{ik})}{Var(Y_{ik})} = Con(Y_{ik}) + Spe(Y_{ik})$$

Konsistenz

$$Kon(Y_{ik}) = \frac{Var(x_{ik})}{Var(Y_{ik})}$$

Meßgelegenheitsspezifität

$$Spe(Y_{ik}) = \frac{Var(z_{ik})}{Var(Y_{ik})}$$

Stabilität der Latent-Statevariablen

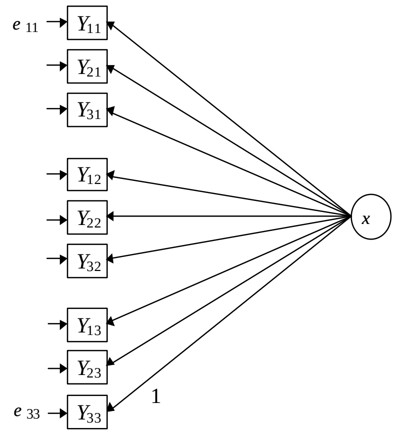
$$Kor(t_{ik}, t_{il})$$

Stabilität der Latent-Traitvariablen

$$Kor(x_{ik}, x_{il})$$

Latent-State-Trait-Theorie

Singletrait-Modell (ohne Methodenfaktoren)



Latent-State-Trait-Theorie

Singletrait-Modell (ohne Methodenfaktoren)

Definition

$$Y_{it} = t_{it} + e_{it}$$

$$= \lambda_{it0} + \lambda_{it1} x + e_{it}$$

$$Cov(e_{it}, e_{js}) = 0 \qquad (i, t) \neq (j, s)$$

Latent-State-Trait-Theorie

Singletrait-Modell (ohne Methodenfaktoren)

Identifikation bei Fixierung der Skala von x *durch*

$\lambda_{110} := 0$ und $\lambda_{111} := 1$

$$E(x) = E(Y_{11})$$

$$Var(x) = \frac{Cov(Y_{11}, Y_{it}) \cdot Cov(Y_{11}, Y_{js})}{Cov(Y_{it}, Y_{js})}, \quad (i,t) \neq (j,s),\ (i,t),(j,s) \neq (1,1)$$

$$\lambda_{it1}^2\, Var(x) = \frac{Cov(Y_{it}, Y_{js}) \cdot Cov(Y_{it}, Y_{ku})}{Cov(Y_{js}, Y_{ku})}, \quad (j,s) \neq (k,u),$$

$$Var(e_{it}) = Var(Y_{it}) - \lambda_{it1}^2\, Var(x)$$

$$Kon(Y_{it}) = Rel(Y_{it}) = \frac{\lambda_{it1}^2\, Var(x)}{Var(Y_{it})}$$

Latent-State-Trait-Theorie

Singletrait-Modell (ohne Methodenfaktoren)

Testbarkeit über die Kovarianzstruktur

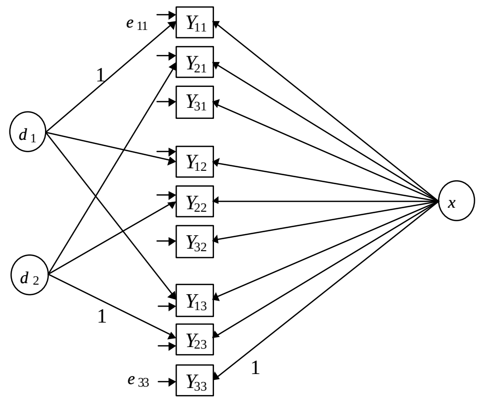
$$Var(Y_{it}) = Var(t_{it}) + Var(e_{it})$$

$$= \lambda_{it1}^2\, Var(x) + Var(e_{it})$$

$$Cov(Y_{it}, Y_{js}) = Cov(t_{it}, t_{js}) = \lambda_{it1} \lambda_{js1} Var(x), \quad (i,t) \neq (i,s)$$

Latent-State-Trait-Theorie

Singletrait Modell (mit Methodenfaktoren)



Latent-State-Trait-Theorie

Singletrait-Modell (mit Methodenfaktoren)

Definition

$$Y_{it} = t_{it} + e_{it}$$

$$= \lambda_{it0} + \lambda_{it1} x + \lambda_{it2} d_i + e_{it}$$

$$E(e_{it}) = 0$$

$$E(d_i) = 0$$

$$Cov(e_{it}, e_{js}) = 0 \quad (i, t) \neq (j, s)$$

$$Cov(e_{it}, d_i) = 0$$

$$Cov(e_{it}, x) = 0$$

$$Cov(x, d_i) = 0$$

Latent-State-Trait-Theorie

Singletrait-Modell (mit Methodenfaktoren)

Identifikation bei Fixierung der Skala von x *durch*

$\lambda_{110} := 0$, $\lambda_{111} := 1$, und $\lambda_{112} := 1$

$$E(x) = E(Y_{11})$$

$$Var(x) = \frac{Cov(Y_{11}, Y_{it}) \cdot Cov(Y_{11}, Y_{js})}{Cov(Y_{it}, Y_{js})}, \quad (i,t) \neq (j,s),\ (i,t),(j,s) \neq$$

$$\lambda_{it1}^2\, Var(x) = \frac{Cov(Y_{it}, Y_{js}) \cdot Cov(Y_{it}, Y_{ku})}{Cov(Y_{js}, Y_{ku})}, \quad (j,s) \neq (k,u),$$

$$Var(e_{it}) = Var(Y_{it}) - \lambda_{it1}^2\, Var(x) - ??$$

$$Kon(Y_{it}) = \frac{\lambda_{it1}^2\, Var(x)}{Var(Y_{it})}$$

$$Rel(Y_{it}) = \frac{\lambda_{it1}^2\, Var(x) + Var(d_i)}{Var(Y\ )}$$

Latent-State-Trait-Theorie

Singletrait-Modell (mit Methodenfaktoren)

Testbarkeit

$$Var(Y_{ik}) = Var(t_{ik}) + Var(e_{ik})$$

$$= \lambda_{ik1}^2\, Var(x) + Var(d_i) + Var(e_{ik})$$

$$Cov(Y_{ik}, Y_{jk}) = Cov(t_{ik}, t_{jk})$$

$$= \lambda_{ik1} \lambda_{jk1}\, Var(x) + Cov(d_i, d_j)$$

$$Cov(Y_{ik}, Y_{il}) = Cov(t_{ik}, t_{il})$$

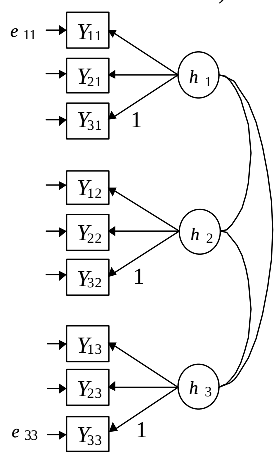
$$= \lambda_{ik1} \lambda_{il1}\, Var(x) + Var(d_i)$$

$$Cov(Y_{ik}, Y_{jl}) = Cov(t_{ik}, t_{jl})$$

$$= \lambda_{ik1} \lambda_{jl1}\, Var(x) + Cov(d_i, d_j)$$

Latent-State-Trait-Theorie

Multistate-Modell (ohne Methodenfaktoren)



Latent-State-Trait-Theorie

Multistate-Modell (ohne Methodenfaktoren)

Definition

$$Y_{ik} = t_{ik} + e_{ik}$$

$$t_{ik} = \lambda_{ik0} + \lambda_{ik1} h_k + e_{ik}$$

$$Cov(e_{ik}, e_{jl}) = 0 \qquad (i, k) \neq (j, l)$$

Identifikation

$$E(h) = E(Y_i)$$

$$Cov(t_{ik}, t_{jl}) = \lambda_{ik1}\lambda_{jl1}\, Cov(h_k, h_l)$$

$$Var(t_{ik}) = \lambda_{ik1}^2\, Var(h_k)$$

$$Var(e_{ik}) = Var(Y_{ik}) - \lambda_{ik1}^2\, Var(h_k)$$

$$Rel(Y_{ik}) = \frac{\lambda_{ik1}^2 Var(h_k)}{Var(Y_{ik})}$$

Latent-State-Trait-Theorie

Singlestate-Modell (ohne Methodenfaktoren)

Testbarkeit

$$Var(Y_{ik}) = Var(t_{ik}) + Var(e_{ik})$$

$$= \lambda_{ik1}^{2} Var(h_k) + Var(e_{ik})$$

$$Cov(Y_{ik}, Y_{jk}) = Cov(t_{ik}, t_{jk})$$

$$= \lambda_{ik1} \lambda_{jk1} Var(h_k)$$

$$Cov(Y_{ik}, Y_{il}) = Cov(t_{ik}, t_{il})$$

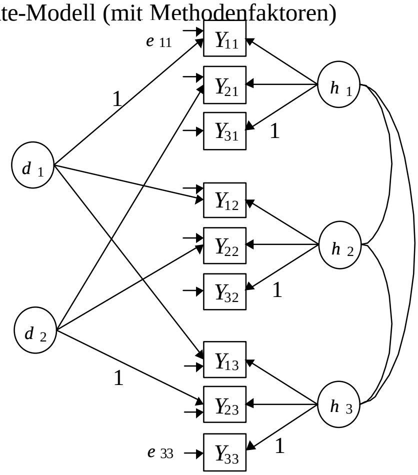
$$= \lambda_{ik1} \lambda_{il1} Cov(h_k, h_l)$$

$$Cov(Y_{ik}, Y_{jl}) = Cov(t_{ik}, t_{jl})$$

$$= \lambda_{ik1} \lambda_{jl1} \mathrm{Cov}(h_k, h_l)$$

Latent-State-Trait-Theorie

Multistate-Modell (mit Methodenfaktoren)



Latent-State-Trait-Theorie

Multistate-Modell (mit Methodenfaktoren)

Definition

$$Y_{ik} = t_{ik} + e_{ik}$$

$$= \lambda_{ik0} + \lambda_{ik1} h_k + d_i + e_{ik}$$

$$h_k = \gamma_{k0} + \gamma_{k1} x + z_k$$

$$Cov(e_{ik}, e_{jl}) = 0 \qquad (i, k) \neq (j, l)$$

$$Cov(e_{ik}, h_l) = 0$$

$$Cov(z_k, z_l) = 0$$

$$Cov(z_k, e_{jl}) = 0$$

$$Cov(z_k, x) = 0$$

Latent-State-Trait-Theorie

Singlestate-Modell (mit Methodenfaktoren)

Identifikation

$$E(h_k) = E(Y_i)$$

$$Cov(t_{ik}, t_{jl}) = \lambda_{ik1} \lambda_{jl1} Cov(h_k, h_l)$$

$$Cov(t_{ik}, t_{jk}) = \lambda_{ik1} \lambda_{jk1} Var(h_k)$$

$$Cov(t_{ik}, t_{il}) = \lambda_{ik1} \lambda_{il1} Cov(h_k, h_l) + Var(d_i)$$

$$Var(t_{ik}) = \lambda_{ik1}^2 Var(h_k) + Var(d_i)$$

$$Var(e_{ik}) = Var(Y_{ik}) - [\lambda_{ik1}^2 Var(h_k) + Var(d_i)]$$

Latent-State-Trait-Theorie

$$Rel(Y_{ik}) = \frac{\lambda_{ik1}^{2} Var(h_k) + Var(d_i)}{Var(Y_{ik})}$$

$$= cRel(Y_{ik}) + mSpe(Y_{ik})$$

$$cRel(Y_{ik}) = \frac{\lambda_{ik1}^{2} Var(h_k)}{Var(Y_{ik})}$$

gemeinsame Reliabilität

$$mSpe(Y_{ik}) = \frac{\lambda_{ik1}^{2} Var(d_i)}{Var(Y_{ik})}$$

Methodenspezifität

Latent-State-Trait-Theorie

Multistate-Modell (mit Methodenfaktoren)

Testbarkeit

$$\mathrm{Var}(Y_{ik}) = \mathrm{Var}(\tau_{ik}) + \mathrm{Var}(\varepsilon_{ik})$$

$$= \lambda_{ik1}^{2} \mathrm{Var}(\eta_k) + \mathrm{Var}(\xi_i) + \mathrm{Var}(\varepsilon_{ik})$$

$$Cov(Y_{ik}, Y_{jk}) = Cov(t_{ik}, t_{jl})$$

$$= \lambda_{ik1} \lambda_{jk1} \mathrm{Var}(\eta_k) + \mathrm{Cov}(\xi_i, \xi_j)$$

$$Cov(Y_{ik}, Y_{il}) = Cov(t_{ik}, t_{il})$$

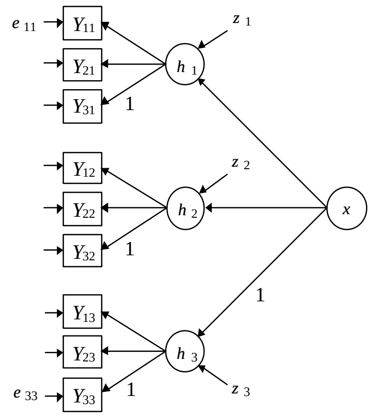
$$= \lambda_{ik1} \lambda_{il1} Cov(h_k, h_l) + Var(x_i)$$

$$Cov(Y_{ik}, Y_{jl}) = Cov(t_{ik}, t_{jl})$$

$$= \lambda_{ik1} \lambda_{jl1} Cov(h_k, h_l) + Cov(x_i, x_j)$$

Latent-State-Trait-Theorie

Singletrait-Multistate Modell mit Methodenfaktoren



Latent-State-Trait-Theorie

Singletrait-Multistate-Modell (ohne Methodenfaktoren)

Definition

$$Y_{ik} = t_{ik} + e_{ik}$$

$$= \lambda_{ik0} + \lambda_{ik1} h_k + e_{ik}$$

$$h_k = \gamma_{k0} + \gamma_{k1} x + z_k$$

$Cov(e_{ik}, e_{jl}) = 0 \quad (i, k) \neq (j, l)$

$Cov(e_{ik}, h_l) = 0$

$Cov(z_k, z_l) = 0$

$Cov(z_k, e_{jl}) = 0$

$Cov(z_k, x) = 0$

Latent-State-Trait-Theorie

Singletrait-Multistate-Modell (ohne Methodenfaktoren)

Identifikation

$$Cov(t_{ik},\ t_{jl}) = Cov(x_{ik},\ x_{jl})$$

$$= \lambda_{ik1}\lambda_{jl1}\gamma_{k1}^{2}\, Var(x\ )$$

$$Cov(t_{ik},\ t_{jk}) = \lambda_{ik1}\lambda_{jk1}\gamma_{k1}^{2}\, Var(x\ )$$

$$+ \lambda_{ik1}\lambda_{jk1}\, Var(z_k)$$

$$Cov(t_{ik},\ t_{il}) = Cov(x_{ik},\ x_{il})$$

$$= \lambda_{ik1}\lambda_{il1}\gamma_{k1}\gamma_{l1}\, Var(x\ )$$

Latent-State-Trait-Theorie

$$Var(t_{ik}) = \lambda_{ik1}^{2}\,\gamma_{k1}^{2}\, Var(x\ ) + \lambda_{ik1}^{2} Var(z_k)$$

$$Var(x_{ik}) = \lambda_{ik1}^{2}\,\gamma_{k1}^{2}\, Var(x\ )$$

$$Var(z_{ik}) = \lambda_{ik1}^{2} Var(z_k)$$

$$Var(e_{ik}) = Var(Y_{ik}) - [\lambda_{ik1}^{2} Var(h_k) + Var(e_{ik})]$$

$$= Var(Y_{ik}) - [\lambda_{ik1}^{2}\,\gamma_{k1}^{2}\, Var(x)$$

$$+ \lambda_{ik1}^{2} Var(z_k)]$$

Latent-State-Trait-Theorie

Singletrait-Multistate-Modell (ohne Methodenfaktoren)

Identifikation

$$Rel(Y_{ik}) = \frac{\lambda_{ik1}^2 \gamma_{k1}^2 Var(x) + \lambda_{ik1}^2 Var(z_k)}{Var(Y_{ik})}$$

$$Kon(Y_{ik}) = \frac{\lambda_{ik1}^2 \gamma_{k1}^2 Var(x)}{Var(Y_{ik})}$$

$$Spe(Y_{ik}) = \frac{\lambda_{ik1}^2 Var(z_k)}{Var(Y_{ik})}$$

Latent-State-Trait-Theorie

Singletrait-Multistate-Modell (ohne Methodenfaktoren)

Testbarkeit

$$\begin{aligned} Var(Y_{ik}) &= Var(t_{ik}) + Var(e_{ik}) \\ &= \lambda_{ik1}^2 \, Var(h_k) + Var(e_{ik}) \\ &= \lambda_{ik1}^2 \, \gamma_{k1}^2 \, Var(x) + \lambda_{ik1}^2 \, Var(z_k) \\ &\quad + Var(e_{ik}) \end{aligned}$$

Latent-State-Trait-Theorie

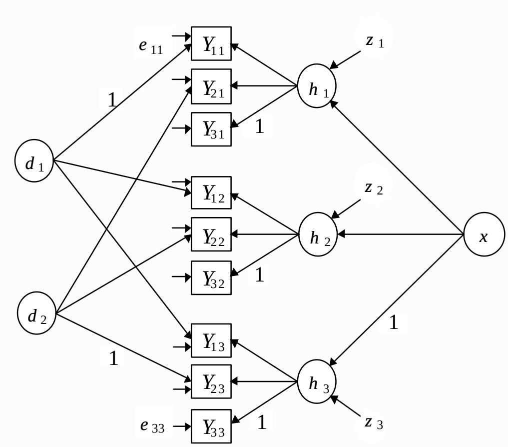
$$Cov(Y_{ik}, Y_{jk}) = Cov(t_{ik}, t_{jk})$$
$$= \lambda_{ik1} \lambda_{jk1} Var(h_k)$$
$$= \lambda_{ik1} \lambda_{jk1} \gamma_{k1}^2 Var(x)$$
$$+ \lambda_{ik1} \lambda_{jk1} Var(z_k)$$

$$Cov(Y_{ik}, Y_{il}) = Cov(t_{ik}, t_{il})$$
$$= \lambda_{ik1} \lambda_{il1} \gamma_{k1} \gamma_{l1} Var(x)$$

$$Cov(Y_{ik}, Y_{jl}) = Cov(t_{ik}, t_{jl})$$
$$= \lambda_{ik1} \lambda_{jl1} \gamma_{k1} \gamma_{l1} Var(x)$$

Latent-State-Trait-Theorie

Singletrait-Multistate-Modell (mit Methodenfaktoren)



Latent-State-Trait-Theorie

Single-Trait-Multi-State-Modelle (mit Methodenfaktoren)

Definition

$$Y_{ik} = t_{ik} + e_{ik}$$

$$= \lambda_{ik0} + \lambda_{ik1} h_k + d_i + e_{ik}$$

$$h_k = \gamma_{k0} + \gamma_{k1} x + z_k$$

$$Cov(e_{ik}, e_{jl}) = 0 \qquad (i, k) \neq (j, l)$$

$$Cov(e_{ik}, h_l) = 0$$

$$Cov(z_k, z_l) = 0$$

$$Cov(z_k, e_{jl}) = 0$$

$$Cov(z_k, x) = 0$$

Latent-State-Trait-Theorie

Single-Trait-Multi-State-Modelle (mit Methodenfaktoren)

Identifikation

$$Cov(t_{ik}, t_{jl}) = Cov(x_{ik}, x_{jl})$$

$$= \lambda_{ik1} \lambda_{jl1} \gamma_{k1} \gamma_{l1} Var(x)$$

$$Cov(t_{ik}, t_{jk}) = \lambda_{ik1} \lambda_{jk1} \gamma_{k1}^2 Var(x)$$

$$+ \lambda_{ik1} \lambda_{jk1} Var(z_k)$$

$$Cov(t_{ik}, t_{il}) = Cov(x_{ik}, x_{il})$$

$$= \lambda_{ik1} \lambda_{il1} \gamma_{k1} \gamma_{l1} Var(x) + Var(d_i)$$

Latent-State-Trait-Theorie

$$Var(t_{ik}) = \lambda_{ik1}^2 \gamma_{k1}^2 Var(x) + \lambda_{ik1}^2 Var(z_k) + Var(x_i)$$

$$Var(x_{ik}) = \lambda_{ik1}^2 \gamma_{k1}^2 Var(x) + Var(d_i)$$

$$Var(z_{ik}) = \lambda_{ik1}^2 Var(z_k)$$

$$Var(e_{ik}) = Var(Y_{ik}) - [\lambda_{ik1}^2 \gamma_{k1}^2 Var(x) + \lambda_{ik1}^2 Var(z_k) + Var(d_i)]$$

Latent-State-Trait-Theorie

Single-Trait-Multi-State-Modelle (mit Methodenfaktoren)

Identifikation

$$Rel(Y_{ik}) = \frac{\lambda_{ik1}^2 \gamma_{k1}^2 Var(x) + \lambda_{ik1}^2 Var(z_k) + Var(d_i)}{Var(Y_{ik})}$$

$$Kon(Y_{ik}) = \frac{\lambda_{ik1}^2 \gamma_{k1}^2 Var(x) + Var(d_i)}{Var(Y_{ik})}$$

$$Spe(Y_{ik}) = \frac{\lambda_{ik1}^2 Var(z_k) + Var(d_i)}{Var(Y_{ik})}$$

Latent-State-Trait-Theorie

Single-Trait-Multi-State-Modell (mit Methodenfaktoren)

Testbarkeit

$$Var(Y_{ik}) = \text{ë}^2_{ik1}\, Var(h_k) + Var(e_{ik})$$

$$= \text{ë}^2_{ik1}\, \text{ã}^2_{k1}\, Var(x) + Var(d_i)$$

$$+ \text{ë}^2_{ik1}\, Var(z_k) + Var(e_{ik})$$

$$Cov(Y_{ik}, Y_{jk}) = Cov(t_{ik}, t_{jk})$$

$$= \lambda_{ik1}\, \lambda_{jk1}\, Var(h_k)$$

$$= \lambda_{ik1}\, \lambda_{jk1}\, \text{ã}^2_{k1}\, Var(x)$$

$$+ \lambda_{ik1}\, \lambda_{jk1}\, Var(z_k)$$

Latent-State-Trait-Theorie

***Single-Trait-Multi-State-Modelle** (mit Methodenfaktoren)*

Testbarkeit (Fortsetzung)

$$Cov(Y_{ik}, Y_{il}) = \lambda_{ik1}\, \lambda_{il1}\, Cov(h_k, h_l)$$

$$= \lambda_{ik1}\, \lambda_{il1}\, \gamma_{k1}\, \gamma_{l1}\, Var(x) + Var(d_i)$$

$$Cov(Y_{ik}, Y_{jl}) = \lambda_{ik1}\, \lambda_{jl1}\, Cov(h_k, h_l)$$

$$= \lambda_{ik1}\, \lambda_{jl1}\, \gamma_{k1}\, \gamma_{l1}\, Var(x)$$