Latent State-Trait Theory

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Basic Concepts and Models of Latent State-Trait Theory

References

Based on:

Steyer, R., Ferring, D., and Schmitt, M. J. (1992).States and Traits in Psychological Assessment. *European Journal of Psychological Assessment*, 8, 79-98.

Steyer, R., Schmitt, M. and Eid, M. (1999). Latent-state-trait theory and research in personality and individual differences. *European Journal of Personality*, 13, 389-408.

Basic Concepts of Latent State-Trait Theory

The set of possible outcome of the random experiment:

$\Omega = \Omega_U \times \Omega_{S_1} \times ... \times \Omega_{S_t} \times ... \times \Omega_{S_n} \times \Omega_{O_1} \times ... \times \Omega_{O_t} \times ...\Omega_{O_n}$

Test-score variables:

$Y_{it} : \Omega \longrightarrow \mathbb{R}$

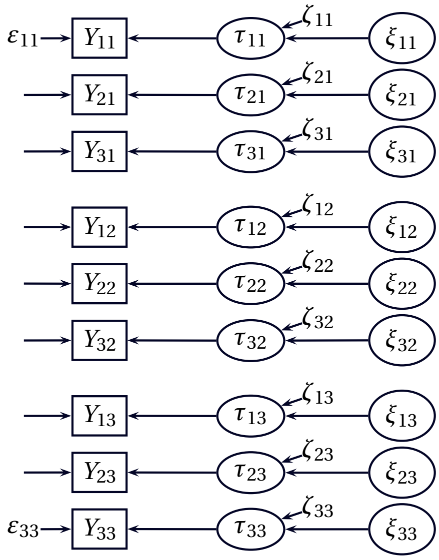
Person- and situation variables:

$U : \Omega \longrightarrow \Omega_U$ person variable
$S_t : \Omega \longrightarrow \Omega_{S_t}$ situation variable

Theoretical Variables

$\tau_{it} := E(Y_{it} \mid U, S_t)$	Latent state variable
$\varepsilon_{it} := Y_{it} - \tau_{it}$	Measurement error variable
$\xi_{it} := E(Y_{it} \mid U)$	Latent trait variable
$\zeta_{it} := \tau_{it} - \xi_{it}$	Latent state residual

Path Diagram



Properties of the Latent Variables

Decomposition of the variables

$Y_{it} = \tau_{it} + \varepsilon_{it}$

$\tau_{it} = \xi_{it} + \zeta_{it}$

Decomposition of the variances

$Var(Y_{it}) = Var(\tau_{it}) + Var(\varepsilon_{it})$

$Var(\tau_{it}) = Var(\xi_{it}) + Var(\zeta_{it})$

Expected Values and Covariances of the Error Variables

Implications of the definitions of the latent state- and latent trait variables:
$E(\varepsilon_{it}) = 0$
$E(\zeta_{it}) = 0$

$Cov(\varepsilon_{it}, \zeta_{jt}) = 0$
$Cov(\varepsilon_{it}, \tau_{jt}) = 0$
$Cov(\varepsilon_{it}, \xi_{js}) = 0$
$Cov(\zeta_{it}, \xi_{js}) = 0$

Assumptions:
$Cov(\varepsilon_{it}, \zeta_{js}) = 0 \qquad s \neq t$

$Cov(\varepsilon_{it}, \tau_{js}) = 0 \qquad s \neq t$

Proof

Proof of $E(\zeta_{it}) = 0$:

$$\begin{aligned} E(\zeta_{it}) &= E(\tau_{it} - \xi_{it}) \\ &= E(\tau_{it}) - E(\xi_{it}) \\ &= E[E(Y_{it} \mid U, S_t)] - E[E(Y_{it} \mid U)] \\ &= E(Y_{it}) - E(Y_{it}) \\ &= 0 \end{aligned}$$

Proof

Proof of: $Cov(\zeta_{it}, \xi_{js}) = 0$.

First, we show that $E(\zeta_{it} \mid \xi_{js}) = 0$, because this implies: $Cov(\zeta_{it}, \xi_{js}) = 0$.

$$\begin{aligned} \zeta_{it} &= \tau_{it} - \xi_{it} \\ &= \tau_{it} - E(Y_{it} \mid U) \\ &= \tau_{it} - E[E(Y_{it} \mid U, S_t) \mid U] \\ &= \tau_{it} - E(\tau_{it} \mid U) \end{aligned}$$

Hence ζ_{it} is a residual with respect to the regressor U. This implies that the regression of ζ_{it} on each (measurable) function of U is equal to 0 and $\xi_{js} := E(Y_{js} \mid U)$ is such a function.

Important Coefficients

Reliability

$$Rel(Y_{it}) = \frac{Var(\tau_{it})}{Var(Y_{it})} = Con(Y_{it}) + Spe(Y_{it})$$

Consistency

$$Con(Y_{it}) = \frac{Var(\xi_{it})}{Var(Y_{it})}$$

Situational specificity

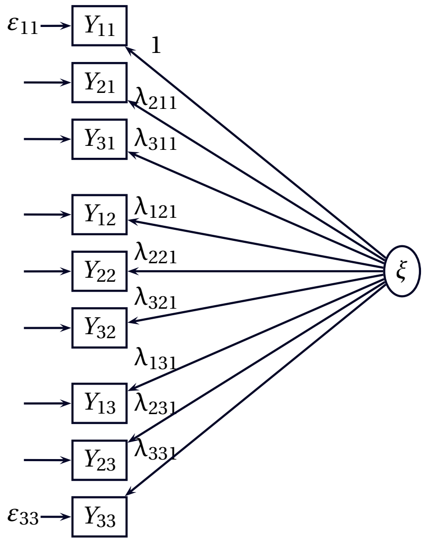
$$Spe(Y_{it}) = \frac{Var(\zeta_{it})}{Var(Y_{it})}$$

Stability of the latent state variable: $Kor(\tau_{it}, \tau_{is})$

Stability of the latent trait variable: $Kor(\xi_{it}, \xi_{is})$

Singletrait Model

Path Diagram of the Singletrait Model



Definition of the Singletrait Model

The following assumptions define the Singletrait Model:

$$\begin{aligned} Y_{it} &= \tau_{it} + \varepsilon_{it} \\ &= \lambda_{it0} + \lambda_{it1}\,\xi + \varepsilon_{it} \end{aligned}$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0 \quad (i,t) \neq (j,s)$$

Fixing Scales and Identification

Fixing the scale of the latent trait variable ξ

$$E(\xi) = 0\,;\;\; Var(\xi) = 1 \tag{1}$$

$$\lambda_{110} = 0\,;\;\; \lambda_{111} = 1 \tag{2}$$

Identification for $\lambda_{110} = 0$ and $\lambda_{111} = 1$:

$E(\xi) = E(Y_{11})$

$\lambda_{it1}^2\, Var(\xi) = \dfrac{Cov(Y_{it}, Y_{js})\; Cov(Y_{it}, Y_{kv})}{Cov(Y_{js}, Y_{kv})} \quad (i,t) \neq (j,s) \neq (k,v)$

$Var(\tau_{it}) = \lambda_{it1}^2\, Var(\xi)$

$Con(Y_{it}) = Rel(Y_{it}) = \dfrac{\lambda_{it1}^2\, Var(\xi)}{Var(Y_{it})}$

Testability

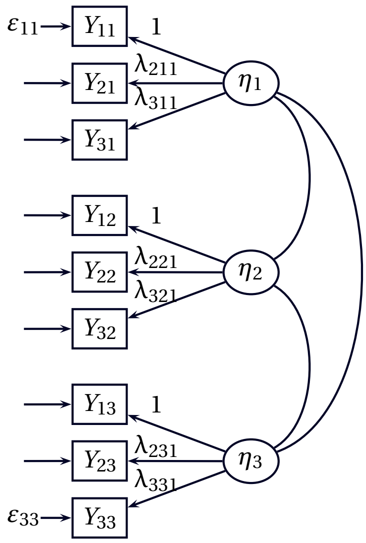
The variance-covariance structure is:

$$\begin{aligned} Var(Y_{it}) &= Var(\tau_{it}) + Var(\varepsilon_{it}) \\ &= \lambda_{it1}^2\, Var(\xi) + Var(\varepsilon_{it}) \end{aligned}$$

$$Cov(Y_{it}, Y_{js}) = Cov(\tau_{it}, \tau_{js}) = \lambda_{it1}\lambda_{js1}\, Var(\xi) \quad (i,t) \neq (j,s)$$

Multistate Model

Path Diagram of the Multistate Model



Definition of the Multistate Model

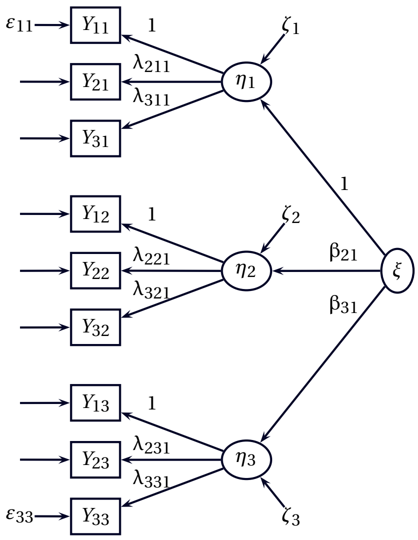
The following assumptions define the Multistate Model:

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$\tau_{it} = \lambda_{it0} + \lambda_{it1}\,\eta_t$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0 \quad (i,t) \neq (j,s)$$

Path Diagram of the Multistate-Singletrait Model



Definition of the Multistate-Singletrait Model

The following assumptions define the Multistate-Singletrait Model:

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$= \lambda_{it0} + \lambda_{it1}\,\eta_t + \varepsilon_{it}$$

$$\eta_t = \beta_{t0} + \beta_{t1}\,\xi + \zeta_t$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0, \quad (i,t) \neq (j,s)$$

$$Cov(\varepsilon_{it}, \eta_s) = 0$$

$$Cov(\zeta_t, \zeta_s) = 0, \quad s \neq t$$

$$Cov(\zeta_t, \varepsilon_{js}) = 0$$

$$Cov(\zeta_t, \xi) = 0$$