# Models of Classical Test Theory

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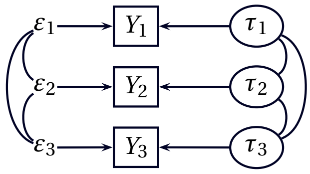
### Where we are






## Model of Parallel Tests



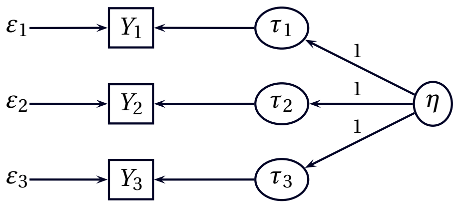
### Model of Parallel Tests

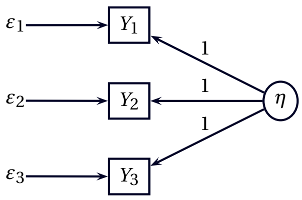
Assumptions ($a_1$), (b), and (c):

- ($a_1$) $\tau$-equivalence $\tau_i = \tau_j$
- (b) uncorrelated errors $Cov(\varepsilon_i, \varepsilon_j) = 0, \quad i \neq j$
- (c) equal error variances $Var(\varepsilon_i) = Var(\varepsilon_j)$



### Path Diagram



## Implied Covariance Structure

$$\begin{aligned} Cov(Y_1, Y_2) &= Cov(\eta + \varepsilon_1, \eta + \varepsilon_2) \\ &= Cov(\eta, \eta) + Cov(\varepsilon_1, \varepsilon_2) + Cov(\eta, \varepsilon_1) + Cov(\eta, \varepsilon_2) \\ &= Var(\eta) \\ &= :\sigma_\eta^2 \end{aligned}$$

$$\begin{aligned} Var(Y_i) - Cov(Y_1, Y_2) &= Var(\varepsilon_i) \\ &=: \sigma_\varepsilon^2 \end{aligned}$$

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## Implied Covariance Matrix

Implied covariance matrix for 3 parallel tests:

$$\begin{bmatrix} \sigma_\eta^2 + \sigma_\varepsilon^2 & \sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\varepsilon^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\varepsilon^2 \end{bmatrix}$$

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## Identification of the Theoretical Parameters

The theoretical parameters can be computed from the empirical parameters as follows:

$$\begin{aligned} E(\eta) &= E(Y_i) \\ Var(\eta) &= Cov(Y_i, Y_j), \quad i \neq j \\ Var(\varepsilon_i) &= Var(Y_i) - Cov(Y_i, Y_j), \quad i \neq j \\ Rel(Y_i) &= Kor(Y_i, Y_j), \quad i \neq j \end{aligned}$$

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## Testability

Testability in the Total Population and in Subpopulations

- $E(Y_i) = \mu$
- $Var(Y_i) = \sigma_Y^2$
- $Cov(Y_i, Y_j) = \sigma_\eta^2 \qquad i \neq j.$

In the total population all expected values are equal, all variances are equal, and all covariances of different variables $Y_i$ are equal.

Testability in each subpopulation

- $E^{(s)}(Y_i) = \mu^{(s)}$
- $Var^{(s)}(Y_i) = \sigma_Y^{2\,(s)}$
- $Cov^{(s)}(Y_i, Y_j) = \sigma_\eta^{2\,(s)} \qquad i \neq j.$

In a subpopulation $s$ all expected values are equal, all variances are equal, and all covariances are equal for different variables $Y_i$. Between *different* subpopulations they can differ.

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## Spearman-Brown Formula

Sum score $S := Y_1 + \ldots + Y_m$

Assumptions ($a_1$), (b) and (c) imply:

Spearman-Brown Formula for parallel tests:

$$Rel(S) = \frac{m \cdot Rel(Y)}{1 + (m-1) \cdot Rel(Y)}.$$

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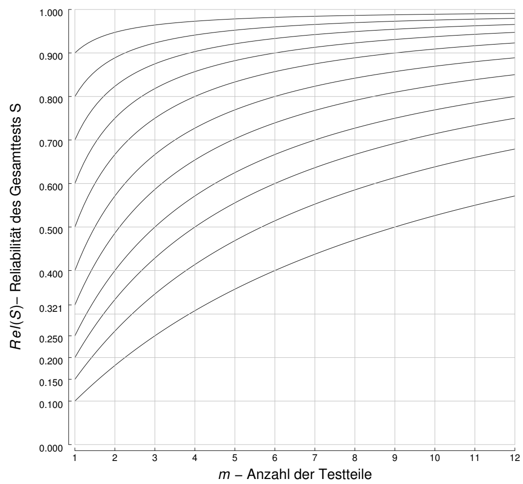
## Spearman-Brown Chart




Grafische Darstellung der Spearman–Brown–Formel

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