

# Introduction Into Latent Variable Modeling With Mplus

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## Overview

- (1) Classical Test Theory (CTT)
  - Basic concepts of CTT
  - Model of parallel tests
  - Model of essential  $\tau$ -equivalent tests
  - Model of  $\tau$ -congeneric tests
  - Applications in Mplus
- (2) Latent State-Trait Theory (LST)
  - Basic concepts of LST
  - Applications in Mplus
- (3) Method Effects

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## Readings on CTT

- Steyer, R. & Eid, M. (2001). *Messen und Testen*. Berlin: Springer.
- Steyer, R., Wolf, A., Funke, E. & Partchev, I. (in press). *Strukturgleichungsmodelle*. In: H. Holling & R. Schwarzer (Hrsg.), *Evaluationsforschung: Band 1: Modelle und Methoden*. (Handbuch der Psychologie). Göttingen: Hogrefe.
- Steyer, R. (1988). Conditional Expectations: An Introduction to the Concept and its Applications in Empirical Sciences. *Methodika*, 2, 53-78.
- Steyer, R. (1989). Models of Classical Psychometric Test Theory as Stochastic Measurement Models: Representation, Uniqueness, Meaningfulness, Identifiability, and Testability. *Methodika*, 3, 25-60.
- Steyer, R. (2001). Classical Test Theory. In: Ragin, C. and Cook, T. (Eds.), *International Encyclopedia of the Social and Behavioural Sciences. Logic of Inquiry and Research Design* (pp. 1955-1962). Pergamon, Oxford.

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## Readings on LST

- Steyer, R., Ferring, D., & Schmitt, M. J. (1992). States and Traits in Psychological Assessment. *European Journal of Psychological Assessment*, 8, 79-98.
- Steyer, R., Schmitt, M. & Eid, M. (1999). Latent state-trait theory and research in personality and individual differences. *European Journal of Personality*, 13, 389-408.

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## Readings on Method Factors

- Pohl, S., Steyer, R. & Kraus, K. (2008). Modelling method effects as individual causal effects. *Journal of the Royal Statistical Society, Series A*, 171, 41-63.
- Pohl, S. & Steyer, R. (submitted). A multitrait-multimethod model with common trait factors.

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## Articles, handouts etc.

- <http://www.metheval.uni-jena.de/courses.php?course=73>

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**Basic Concepts of Classical Test Theory**

Primitives:

- The set of possible outcomes of the random experiment  $\Omega = \Omega_U \times \Omega_O$
- Test-score variables  $Y_i : \Omega \rightarrow \mathbb{R}$
- Person mapping  $U : \Omega \rightarrow \Omega_U$   
where  $U(\omega) = u, \quad u \in \Omega_U$

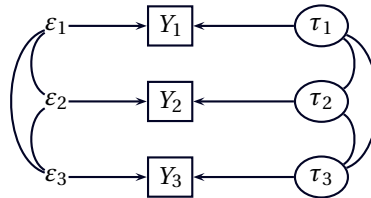
Basic concepts (definition):

- True-score variables  $\tau_i := E(Y_i | U)$
- Measurement error variables  $\varepsilon_i := Y_i - E(Y_i | U)$

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**Path diagram**



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**Basic Concepts of Classical Test Theory**

Properties of true-score- and measurement error variables implied by their definition:

Decomposition of variables

$$Y_i = \tau_i + \varepsilon_i \tag{1}$$

Decomposition of variances

$$Var(Y_i) = Var(\tau_i) + Var(\varepsilon_i) \tag{2}$$

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**Basic Concepts of Classical Test Theory**

More properties of true-score and measurement error variables implied by their definition:

$$Cov(\tau_i, \varepsilon_j) = 0 \tag{3}$$

$$E(\varepsilon_i) = 0 \tag{4}$$

$$E(\varepsilon_i | U) = 0. \tag{5}$$

For each mapping  $f(U)$  of  $U$ :

$$E[\varepsilon_i | f(U)] = 0. \tag{6}$$

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## Basic Concepts of Classical Test Theory

Reliability parameters

Reliability  $Rel(Y_i) := \frac{Var(\tau_i)}{Var(Y_i)}$  Parameter for global reliability of  $Y_i$

error variance  $Var(\varepsilon_i)$  Parameter for global unreliability of  $Y_i$

conditional error variance  $Var(\varepsilon_i | U=u)$  Parameter for local unreliability of  $Y_i$

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## Models of Classical Test Theory

Typical assumptions in models of Classical Test Theory

(a<sub>1</sub>)  $\tau$ -equivalence  $\tau_i = \tau_j$

(a<sub>2</sub>) essential  $\tau$ -equivalence  $\tau_i = \tau_j + \lambda_{ij}$ ,  
 $\lambda_{ij} \in \mathbb{R}$

(a<sub>3</sub>)  $\tau$ -congenericity  $\tau_i = \lambda_{ij0} + \lambda_{ij1}\tau_j$ ,  
 $\lambda_{ij0}, \lambda_{ij1} \in \mathbb{R}, \lambda_{ij1} > 0$

(b) uncorrelated measurement errors  $Cov(\varepsilon_i, \varepsilon_j) = 0, i \neq j$

(c) equal measurement error variances  $Var(\varepsilon_i) = Var(\varepsilon_j)$

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## Models of Classical Test Theory

Models defined by the assumptions:

- Model of parallel tests: (a<sub>1</sub>), (b) und (c)
- Model of  $\tau$ -equivalent tests: (a<sub>2</sub>) und (b)
- Model of  $\tau$ -congeneric tests: (a<sub>3</sub>) und (b)

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