

Basic Concepts of Statistical Inference for Causal Effects in Experiments and Observational Studies

I. Framework

1. Primitives: Units, treatments, potential outcomes
 - comparisons of potential outcomes define causal effects
 - other uses of “cause”
 - the causal effect of one treatment vs. another on one unit
 - covariates – help predict missing potential outcome
 - relating this definition to common usage of “cause”
 - relationship to the “but-for” concept in legal settings
2. Learning about causal effects: Replication, stability, the assignment mechanism
 - replication of units
 - complications with more than one unit – notation
 - interference between units
 - even more complication – variation of treatment effectiveness
 - simplification under stability – SUTVA
 - inference for causal effects is a statistical problem involving missing data created by an assignment mechanism
 - illustrating the criticality of the assignment mechanism
 - what if the doctor has randomized?
 - Lord’s paradox
3. The transition to statistical inference: introduction to randomized experiments and the Rubin Causal Model
 - Neyman (1923) notation in randomized experiments
 - Fisher’s (1925) proposal to physically randomize
 - potential outcomes in nonrandomized studies
 - the Rubin Causal Model
 - the assignment mechanism in randomized experiments
 - confounded assignment mechanisms and the formal benefit of randomization
 - formal notation – $P(W|X, Y(0), Y(1))$
 - known unconfounded – probabilities of all assignments known
 - known ignorable – probability of observed assignment known
 - general unconfounded – probabilities known to be function of observed covariates
 - general ignorable – probability of observed assignment known to be function of observed values

II. Causal inference based on the assignment mechanism

4. “Fisherian” significance levels in CR experiment
 - review of completely randomized experiment
 - mathematicians proof by contradiction – irrationality of $\sqrt{2}$,
 - “stochastic” proof by contradiction – p-values, significance
 - a priori choices – null hypothesis, statistic, direction
 - “confidence intervals” for “additive” effects
 - transformations to additivity
 - efficiency of different statistics – gain scores
 - importance of conditioning on sizes of treatment groups
5. “Neymanian” repeated sampling evaluations in CR experiment
 - unbiasedness of sample means for population means
 - evaluation of interval created under additivity in large studies
 - conservativeness of additive interval in large samples
 - generalization of inference to larger larger populations
 - general repeated sampling evaluation of any procedure
 - efficiency considerations and covariates
6. Extension to studies with variable but known propensities
 - discrete covariate groups – different “weights”
 - probability of a unit being treated – propensity scores
 - unbiased estimates – weighting and HT estimation
 - warning about variance of unbiased estimates – Basu’s elephant
 - need for overlap in propensities
 - Fisherian p-values and interval estimation
7. Extension to studies with unknown propensities
 - one discrete covariate
 - a few discrete covariates
 - many covariates – warning about overlap
 - examples of observational studies with posited assignment mechanism
 - sensitivity to unknown assignment mechanism assumed unconfounded – need for balance in covariates possibly unconfounded
 - bounds = special case of sensitivity

III. Causal inference based on predictive distributions of potential outcomes

8. Predictive inference – intuition under ignorability

- only observed covariates need be considered
- extra efficiency possible – see with Fisherian and Neymanian modes
- multiple imputation of missing potential outcomes
- donor pools with one discrete covariate – matching
- donor pools with several discrete covariates – matching
- donor pools with one continuous covariate – edge effects
- creating donor pools of unobserved values – bridge to models
- donor pools with many covariates – propensity scores
- creating “enhanced” donor pools
- clear role of overlap in distributions with predictive
- examples of posterior predictive distributions

9. Formal predictive inference – Bayesian [Rubin,1978]

- supplements $P(W|X, Y)$ with $P(Y|X)$ and $P(X)$
- CR assignment mechanism – ignorable
- CR derivation of posterior distribution
- model for data critical – used for prediction
- posterior predictive distribution of missing data
- prescriptive for any situation
- sensitivity to model specification

10. “Mixed” modes of inference

- find Bayes estimate – do Fisher p-value
- find Bayes interval. – do Neyman evaluations
- can be truly mixed when there exist complications

11. Examples when reliance on predictive approach is needed

- missing outcomes in RCT
- noncompliance in RCT and want effect of treatment not assignment – instrumental variables approach
- other principal stratification examples
- IV in special cases

12. Assessment of critical assumptions

- overlap in distributions under ignorability
- justification for ignorability
- rationale for various nonignorable exclusion restrictions