Individual causal effect models

Based on the following papers

Steyer, R. (First draft). Analyzing Individual and Average Causal Effects via Structural Equation Models.

Steyer, R., Gabler, S., von Davier, A., Nachtigall, C. & Buhl, T. (2000a) Causal re-gres-sion models I: individual and average causal effects. Methods of Psychological Research-Online, 5, 2, 39-71. (http://www.mpr-online.de)

Steyer, R., Gabler, S., von Davier, A. & Nachtigall, C. (2000b) Causal regression models II: unconfoundedness and causal unbiasedness. Methods of Psychological Research-Online, 5, 3, 55-86. (http://www.mpr-online.de)

Abstract

A design and a method of data analysis is presented which yield estimates of

(a) the *average causal effect* of a treatment variable on a response variable in the sense of Rubins approach to causality

(b) the *variance of the individual causal effects*

(c) the *individual causal effects* themselves and

(d) of the *covariance between pretest and individual causal effects*.

(e) It is shown how to include variables in the analysis that *explain the interindividual differences in the individual causal effects* of the treatment variable on the response variable.

Basic concepts: expected outcomes, individual and average causal effects

Table 1. *Individual causal effects, equal and unequal treatment probabilities.*

$$E(Y \mid X = x) := \sum_u E(Y \mid X = x, U = u) \cdot P(U = u \mid X = x)$$

Person	$P(U{=}u)$ sampling probability	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effect	$P(X = 1 \mid U{=}u)$ treatment probability in experiment 1	$P(X = 1 \mid U{=}u)$ treatment probability in experiment 2
u_1	1/8	82	68	14	1/2	8/9
u_2	1/8	89	81	8	1/2	7/9
u_3	1/8	101	89	12	1/2	6/9
u_4	1/8	108	102	6	1/2	5/9
u_5	1/8	118	112	6	1/2	4/9
u_6	1/8	131	119	12	1/2	3/9
u_7	1/8	139	131	8	1/2	2/9
u_8	1/8	152	138	14	1/2	1/9
		Individual causal laws			Good design	Bad design

Causally unbiased expectation:

$$CUE(Y \mid X = x) = \sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$$

Good design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is average causal effect

Bad design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is not the average causal effect

1. The Single-Unit Trial

We consider the following *single-unit trial*: sample a unit (or person) from a given set of units (the population), observe its assignment (or assign it) to one of two treatment conditions and register the outcome.

The set of possible outcomes of the single-unit trial described above might be of the form:

$$\Omega = \Omega_U \times \Omega_X \times \mathrm{IR} .$$

Notation:

- $U: \Omega \rightarrow \Omega_U$ *person variable* or *unit variable*
- $X: \Omega \rightarrow \Omega_X$ *treatment variable*, with values 0 and 1
- $Y: \Omega \rightarrow \mathrm{IR}$ *outcome variable*

2. Individual and Average Causal Effects I

Define the random variables $f_0(U)\colon \Omega \rightarrow \mathrm{IR}$ and $f_1(U)\colon \Omega \rightarrow \mathrm{IR}$ by:

$$f_0(u) = E(Y \mid X = 0, U = u)$$

for all values u of U

and

$$f_1(u) = E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$$

for all values u of U

Then:

$$E(Y \mid X, U) = f_0(U) + f_1(U) \cdot X$$

2. Individual and Average Causal Effects II

Table 1. *Individual causal effects, equal and unequal treatment probabilities.*

$E(Y \mid X = x) := \sum_u E(Y \mid X = x, U = u) \cdot P(U = u \mid X = x)$

Person	$P(U=u)$ sampling probability	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) -$ $E(Y \mid X = 0, U = u)$ Individual causal effec	$P(X = 1 \mid U=u)$ treatment probability in experiment 1	$P(X = 1 \mid U=u)$ treatment probability in experiment 2
u_1	1/8	82	68	14	1/2	8/9
u_2	1/8	89	81	8	1/2	7/9
u_3	1/8	101	89	12	1/2	6/9
u_4	1/8	108	102	6	1/2	5/9
u_5	1/8	118	112	6	1/2	4/9
u_6	1/8	131	119	12	1/2	3/9
u_7	1/8	139	131	8	1/2	2/9
u_8	1/8	152	138	14	1/2	1/9
		Individual causal laws			Good design	Bad design

Causally unbiased expectation:

$CUE(Y \mid X = x) = \sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$

Good design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is average causal effect

Bad design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is not the average causal effect

Define the random variables

$f_0(U)\colon \Omega \rightarrow \mathrm{IR}$ and $f_1(U)\colon \Omega \rightarrow \mathrm{IR}$ by:

$f_0(u) = E(Y \mid X = 0, U = u)$

for all values u of U and

$f_1(u) = E(Y \mid X = 1, U = u)$
$\quad - E(Y \mid X = 0, U = u)$

for all values u of U

Then:

$E(Y \mid X, U) = f_0(U) + f_1(U) \cdot X$

2. Individual and Average Causal Effects III

Definition 1

- $f_1(u) =:$ *Individual causal effect of unit u*
- $E[f_1(U)] =:$ *Average causal effect*

- $f_0(U) =: \eta_0$ *Expected outcome variable under "control"*
- $f_0(U) + f_1(U) =: \eta_1$ *Expected outcome variable under "treatment"*

These two variables, η_0 and η_1, replace Rubins "potential outcome variables".

2. Individual and Average Causal Effects IV

Table 1. *Individual causal effects, equal and unequal treatment probabilities.*

$$E(Y \mid X = x) := \sum_u E(Y \mid X = x, U = u) \cdot P(U = u \mid X = x)$$

Person	$P(U=u)$ sampling probability	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effec	$P(X = 1 \mid U=u)$ treatment probability in experiment 1	$P(X = 1 \mid U=u)$ treatment probability in experiment 2
u_1	1/8	82	68	14	1/2	8/9
u_2	1/8	89	81	8	1/2	7/9
u_3	1/8	101	89	12	1/2	6/9
u_4	1/8	108	102	6	1/2	5/9
u_5	1/8	118	112	6	1/2	4/9
u_6	1/8	131	119	12	1/2	3/9
u_7	1/8	139	131	8	1/2	2/9
u_8	1/8	152	138	14	1/2	1/9
		Individual causal laws			Good design	Bad design

Causally unbiased expectation:

$$CUE(Y \mid X = x) = \sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$$

Good design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is average causal effect

Bad design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is not the average causal effect

Definition 1

- $f_1(u)$ =: *Individual causal effect of unit u*
- $E[\, f_1(U)]$ =: *Average causal effect*

- $f_0(U)$ =: η_0 *Expected outcome variable under "control"*
- $f_0(U) + f_1(U)$ =: η_1 *Expected outcome variable under "treatment"*

These two variables, η_0 and η_1, replace Rubins "potential outcome variables".

Causal Unbiasedness

Definition 2. The regression $E(Y \mid X)$ as well as its values $E(Y \mid X = 0)$ and $E(Y \mid X = 1)$ are called *causally unbiased* if the following equations hold:

$$E(Y \mid X = 0) = E\,[f_0(U)]$$

$$E(Y \mid X = 1) = E\,[f_0(U) + f_1(U)]$$

Corollary 1. If $E(Y \mid X)$ is causally unbiased, then:

$$ACE = E\,[f_1(U)] = E(Y \mid X = 1) - E(Y \mid X = 0)$$

2. Individual and Average Causal Effects IV

Table 1. *Individual causal effects, equal and unequal treatment probabilities.*

$$E(Y \mid X = x) := \sum_u E(Y \mid X = x, U = u) \cdot P(U = u \mid X = x)$$

Person	$P(U=u)$ sampling probability	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u)$ – $E(Y \mid X = 0, U = u)$ Individual causal effec	$P(X = 1 \mid U=u)$ treatment probability in experiment 1	$P(X = 1 \mid U=u)$ treatment probability in experiment 2
u_1	1/8	82	68	14	1/2	8/9
u_2	1/8	89	81	8	1/2	7/9
u_3	1/8	101	89	12	1/2	6/9
u_4	1/8	108	102	6	1/2	5/9
u_5	1/8	118	112	6	1/2	4/9
u_6	1/8	131	119	12	1/2	3/9
u_7	1/8	139	131	8	1/2	2/9
u_8	1/8	152	138	14	1/2	1/9
		Individual causal laws			Good design	Bad design

Causally unbiased expectation:

$$CUE(Y \mid X = x) = \sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$$

Good design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is average causal effect

Bad design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is not the average causal effect

Definition 2. The regression $E(Y \mid X)$ as well as its values $E(Y \mid X = 0)$ and $E(Y \mid X = 1)$ are called *causally unbiased* if the following equations hold:

$$E(Y \mid X = 0) = E\,[f_0(U)]$$

$$E(Y \mid X = 1) = E\,[f_0(U) + f_1(U)]$$

Corollary 1. If $E(Y \mid X)$ is causally unbiased, then:

$$ACE = E\,[f_1(U)]$$

$$= E(Y \mid X = 1) - E(Y \mid X = 0)$$

Identification of the Average Causal Effect I

The following equations are always true (X with values 0 and 1):

$$E(Y \mid X) = \alpha_0 + \alpha_1 \cdot X$$

and

$$E(Y \mid X) = E\,[E(Y \mid X, U) \mid X\,]$$
$$= E\,[\,f_0(U) \mid X\,] + E\,[\,f_1(U) \mid X\,] \cdot X\,.$$

These equations show that the slope α_1 of the linear regression $E(Y \mid X) = \alpha_0 + \alpha_1 \cdot X$ is the average causal effect if

$$E\,[\,f_0(U) \mid X\,] = E\,[\,f_0(U)]$$ Weak

and

$$E\,[\,f_1(U) \mid X\,] = E\,[\,f_1(U)]$$ Ignorability

Identification of the Average Causal Effect Ia

$$E[\, f_0(U) \mid X \,] = E[\, f_0(U)]$$ Weak

and

$$E[\, f_1(U) \mid X \,] = E[\, f_1(U)]$$ Ignorability

$$P[X = 1 \mid f_0(U),\ f_1(U)] = P(X = 1)$$

Identification of the Average Causal Effect II

Theorem 1. Weak ignorability implies *causal unbiasedness* of the regression $E(Y \mid X)$ and, therefore,

$$ACE = E(Y \mid X = 1) - E(Y \mid X = 0) = \alpha_1$$

Theorem 2. Sufficient conditions for weak ignorability and, therefore, for causal unbiasedness are:

- Independence of U and X
- Unit-treatment homogeneity: $E(Y \mid X, U) = E(Y \mid X)$
- Independence of X and (η_0, η_1) (= Rubins Ignorability)
- Unconfoundedness of the regression $E(Y \mid X)$ (see Def. 3)

Identification of the Average Causal Effect III

Definition 3. The regression $E(Y \mid X)$ is called *unconfounded* if for each value x of X:

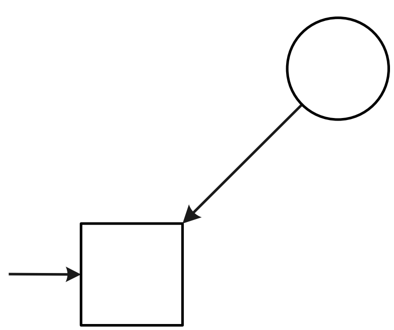
$$P(X = x \mid U = u) = P(X = x), \quad \text{for all values } u \text{ of } U,$$

or

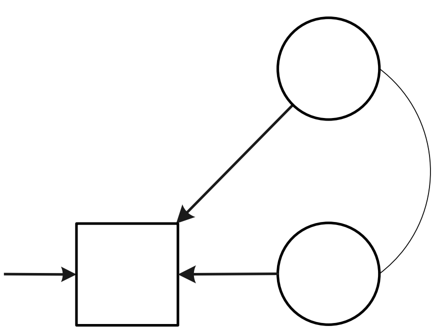
$$E(Y \mid X = x, U = u) = E(Y \mid X = x), \quad \text{for all values } u \text{ of } U.$$

Not Yet Identified Individual Effects Model

0



1



Not Yet Identified Individual Effects Model

Table 1. *Individual causal effects, equal and unequal treatment probabilities.*

$E(Y \mid X = x) := \sum_u E(Y \mid X = x, U = u) \cdot P(U = u \mid X = x)$

Person	$P(U=u)$ sampling probability	$E(Y \mid X = 1, U = u)$ Expected outcome	$E(Y \mid X = 0, U = u)$ Expected outcome	$E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effect	$P(X = 1 \mid U=u)$ treatment probability in experiment 1	$P(X = 1 \mid U=u)$ treatment probability in experiment 2
u_1	1/8	82	68	14	1/2	8/9
u_2	1/8	89	81	8	1/2	7/9
u_3	1/8	101	89	12	1/2	6/9
u_4	1/8	108	102	6	1/2	5/9
u_5	1/8	118	112	6	1/2	4/9
u_6	1/8	131	119	12	1/2	3/9
u_7	1/8	139	131	8	1/2	2/9
u_8	1/8	152	138	14	1/2	1/9
		Individual causal laws			Good design	Bad design

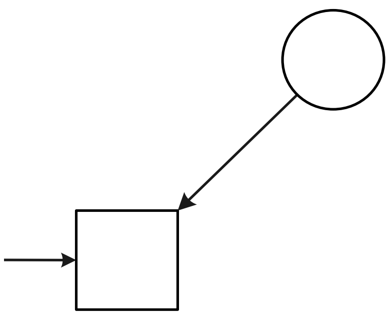
Causally unbiased expectation:

$CUE(Y \mid X = x) = \sum_u E(Y \mid X = x, U = u) \cdot P(U = u)$

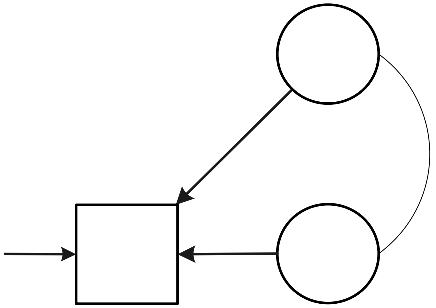
Good design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is average causal effect

Bad design: $E(Y \mid X = 1) - E(Y \mid X = 0)$ is not the average causal effect

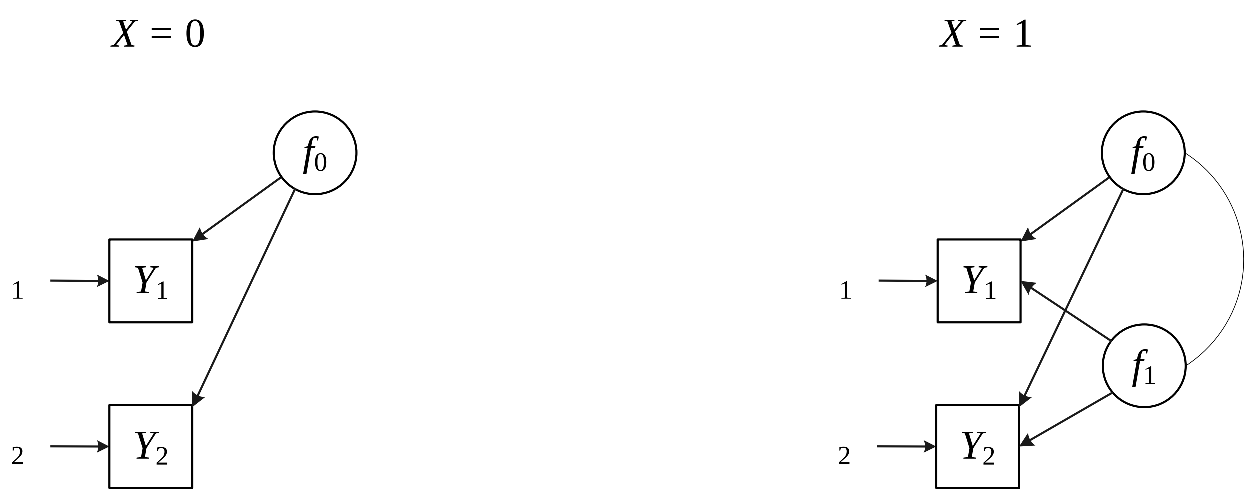
0



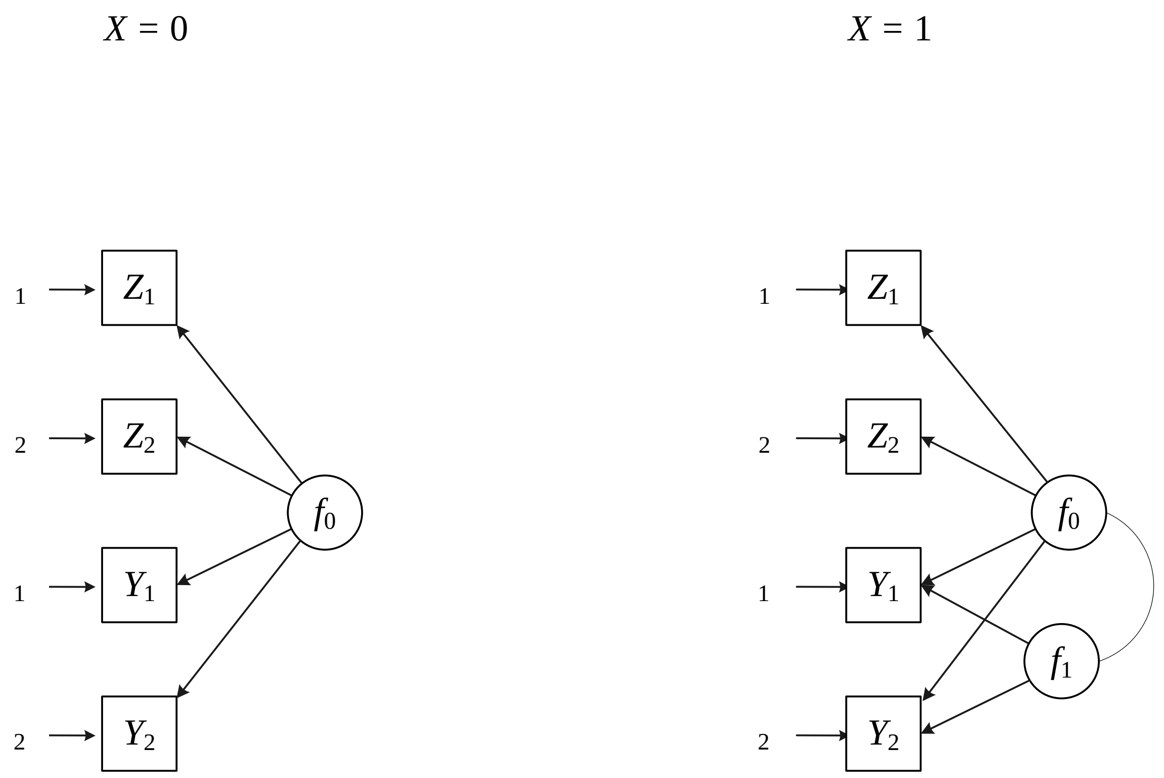
1

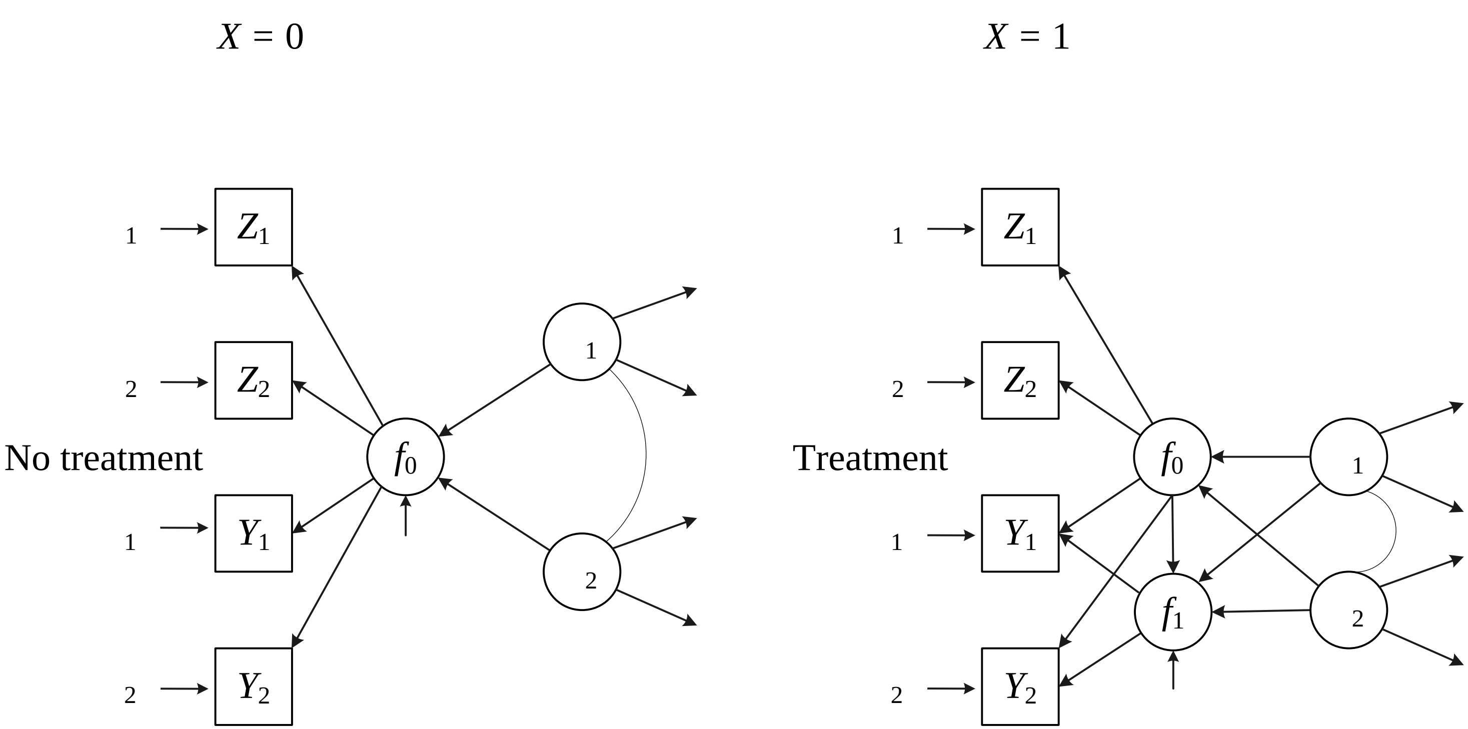


Not Yet Identified Individual Effects Model



Identified Individual Effects Model




X = 0
X = 1
No treatment
Treatment
Z_1
Z_2
Y_1
Y_2
f_0
f_1
1
2

Additional Constraints from Randomization

So far, we did not use the assumption of *U* and *X* being independent which could be created via random assignment of the unit to one of the treatment conditions. We do not need this assumption, because the person in the treatment condition serves, via the pretest, as her own control. This is what the assumptions discussed above are about.

The only assumption we need is that each unit *u* has a positive probability of being assigned to the treatment condition. Otherwise one could argue that there are different processes for the units that have a positive probability of being assigned to the treatment condition as compared to the units that would be assigned to the control condition with probability one. In such a case we would not be able to infer that f_1 is the individual causal effect variable.

If we additionally can assume that X and U are independent, a number of additional testable consequences follow. *First*, the expected values of the individual expected outcome variables in treatment and control conditions are identical:

$$E_0(f_0) = E_1(f_0).$$

Second, the variances of the individual expected outcome variables in the control and treatment conditions are identical:

$$Var_0(f_0) = Var_1(f_0).$$

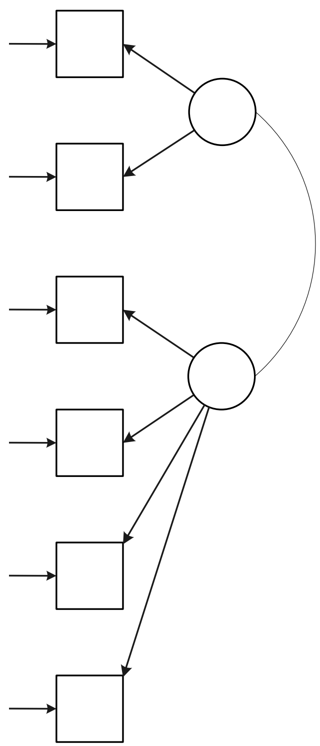
Third, the variances of the measurement error variables of the pretests in the treatment and control conditions are identical:

$$Var_0(\delta_i) = Var_1(\delta_j), \quad i, j = 1, 2.$$

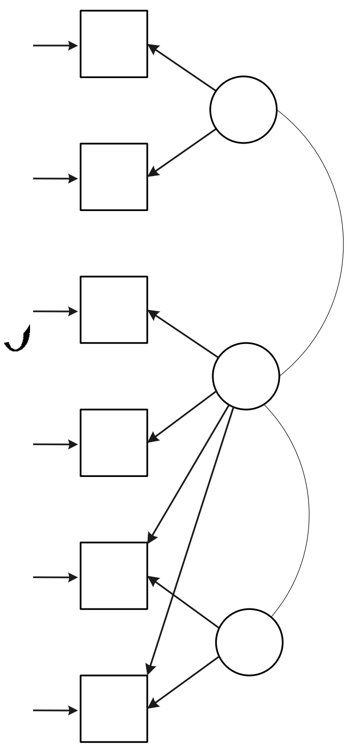
Finally, we also expect the equality of the variances of the measurement error variables of the posttests in the treatment and control conditions:

$$Var_0(\varepsilon_i) = Var_1(\varepsilon_j), \quad i, j = 1, 2.$$

Note, however, that there might also be an effect of the treatment on the measurement error variance. Hence, the last equality is *not* a logical consequence of randomization.

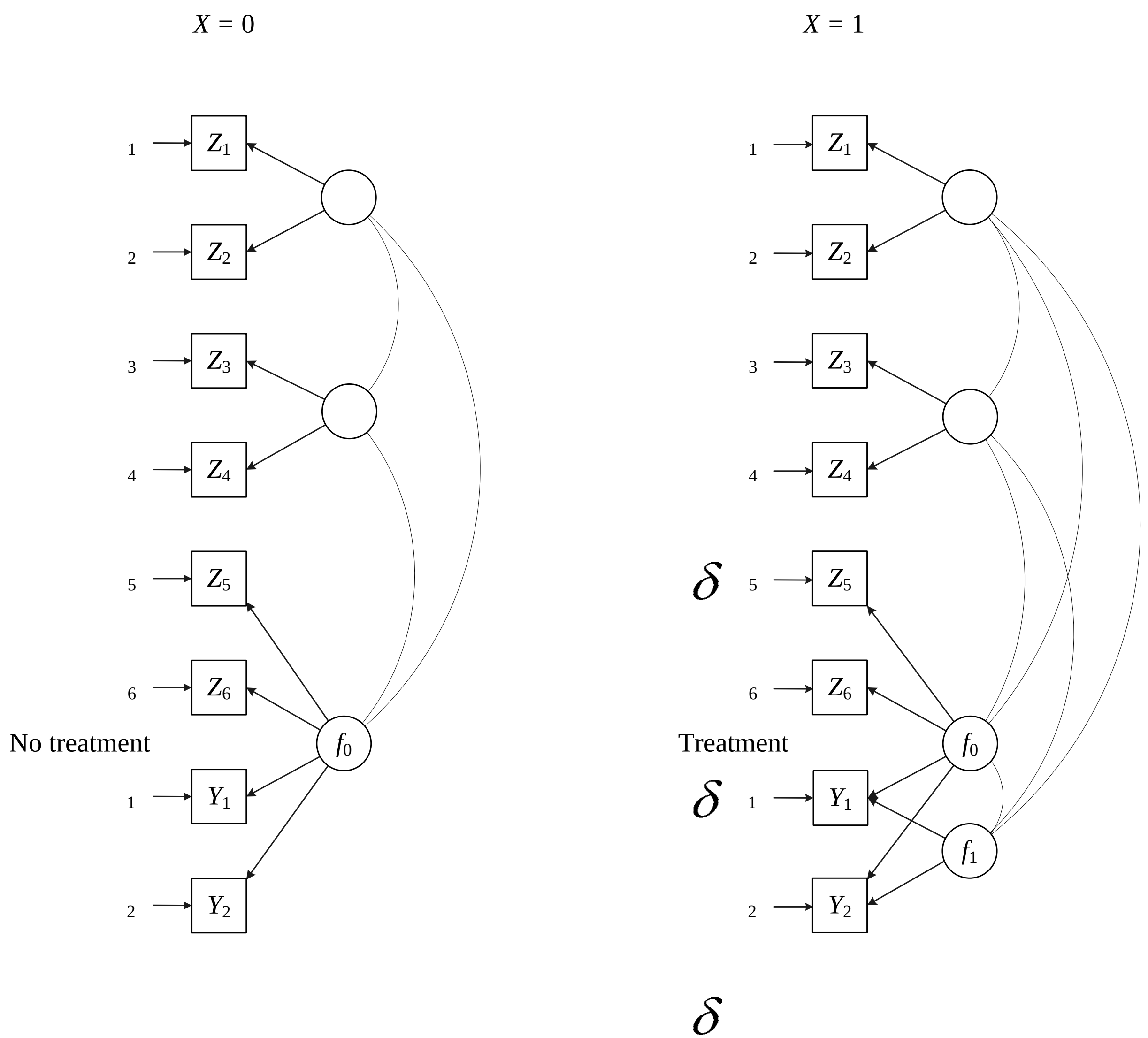


$X =$

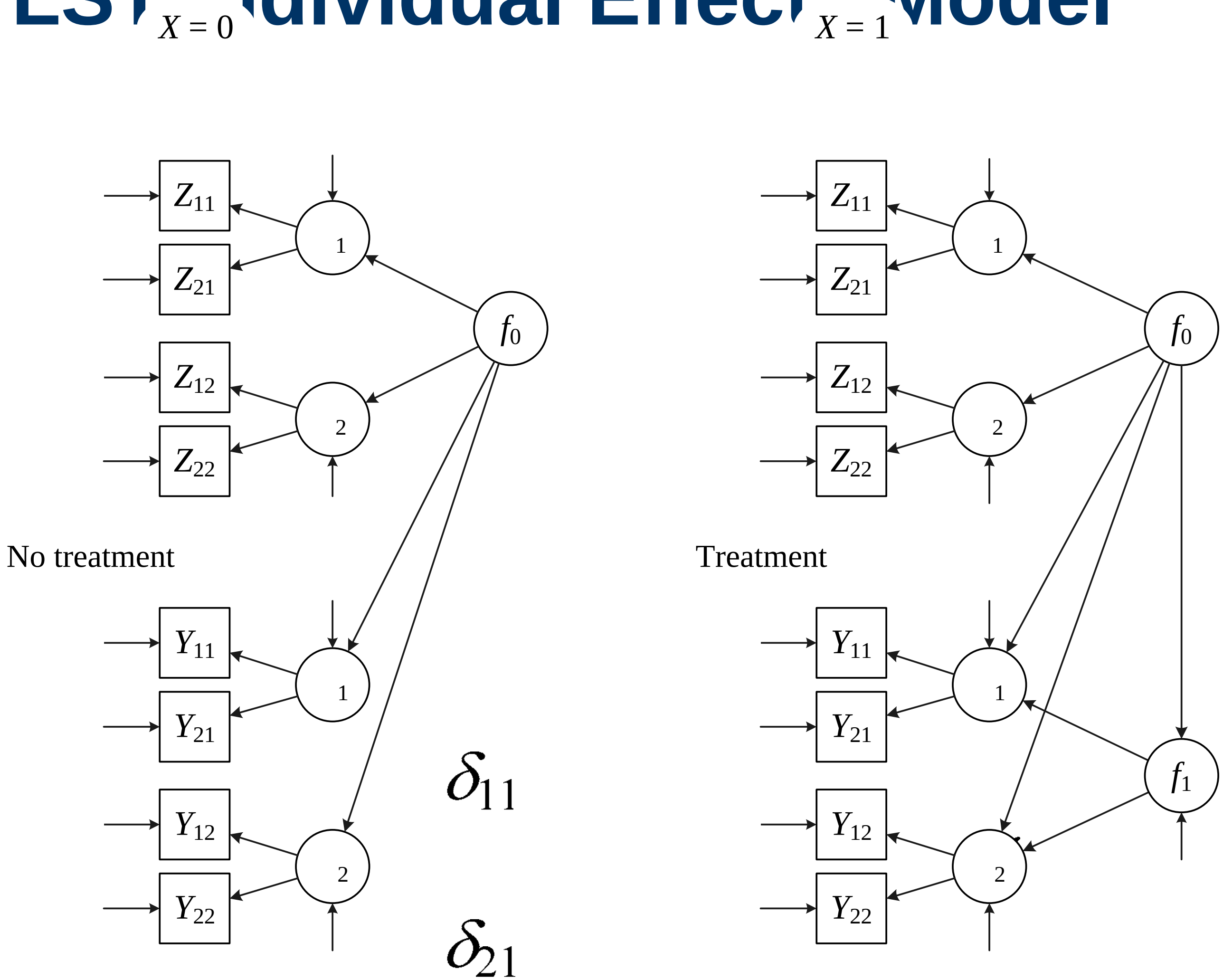


δ_1 Z_1

δ_2 Z_2


X = 0
X = 1
Z1
Z2
Z3
Z4
Z5
Z6
Y1
Y2
f0
f1
No treatment
Treatment
δ

LST Individual Effects Model



LST Individual Effects Model With Method Factor