Latent Growth Curve Models

- Basic ideas
- Time coding 0, 1, 2
- Time coding –1, 0, 1
- Individual trajectories and their growth curve components
- Conclusions

Latent Growth Curve Models

Based on unpublished material of Rolf Steyer

A good paper on Growth Curve Models is:

Tisak, J. & Tisak, M. S. (2000). Permanency and ephemerality of psychological measures with application to organizational commitment. *Psychological Methods*, *5*, 175-198.

For 3 time points, each state η_{ut} of a person u at time t may be computed as a quadratic function of time with the person-specific coefficients π_{u0}, π_{u1}, and π_{u2}:

$$\eta_{ut} = \pi_{u0} + \pi_{u1} \cdot t + \pi_{u2} \cdot t^2$$

For 3 time points t_1, t_2, and t_3 this yields the 3 equations:

$$\eta_{u1} = \pi_{u0} + \pi_{u1} \cdot t_1 + \pi_{u2} \cdot t_1^2$$

$$\eta_{u2} = \pi_{u0} + \pi_{u1} \cdot t_2 + \pi_{u2} \cdot t_2^2$$

$$\eta_{u3} = \pi_{u0} + \pi_{u1} \cdot t_3 + \pi_{u2} \cdot t_3^2$$

If we assume a random experiment in which the person is drawn from a population, the person-specific coefficients π_{u0}, π_{u1}, and π_{u2} turn into values of 3 random variables π_0, π_1, and π_2. The equation for the state variable η_t at time t may then be written:

$$\eta_t = \pi_0 + \pi_1 \cdot t + \pi_2 \cdot t^2$$

For 3 time points t_1, t_2, and t_3 this yields the 3 equations:

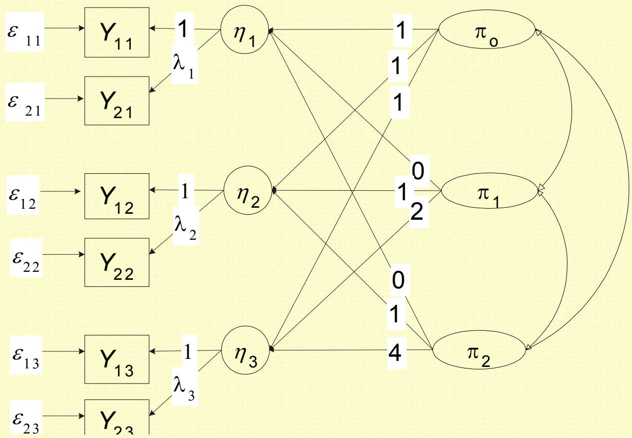
$$\eta_1 = \pi_0 + \pi_1 \cdot t_1 + \pi_2 \cdot t_1^2$$

$$\eta_2 = \pi_0 + \pi_1 \cdot t_2 + \pi_2 \cdot t_2^2$$

$$\eta_3 = \pi_0 + \pi_1 \cdot t_3 + \pi_2 \cdot t_3^2$$

Growth curve model
time coding 0, 1, 2

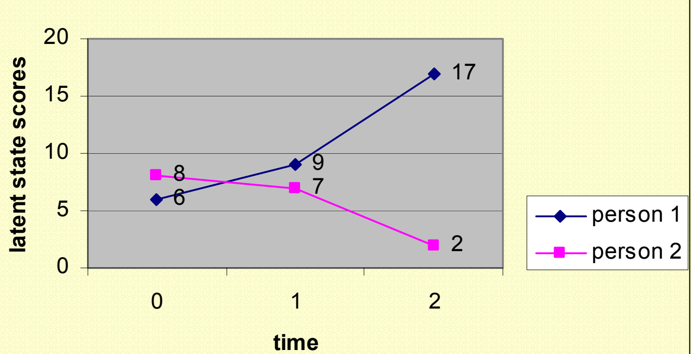
Choosing time scoring $t_1 = 0$, $t_2 = 1$, and $t_3 = 2$, this yields:



with *initial* := π_0, *linear* := π_1, and *quadratic* := π_2.

Growth curve model
time coding 0, 1, 2

Latent state trajectories of 2 persons
time coding 0, 1, 2



Growth curve model
time coding 0, 1, 2

Exercise 1 - time coding 0, 1, 2

The latent state scores η_1, η_2 and η_3 can be translated into growth curve component scores by solving the following 3 equations:

$$\eta_1 = initial$$

$$\eta_2 = initial + linear + quadratic$$

$$\eta_3 = initial + 2 \cdot linear + 4 \cdot quadratic \, .$$

Compute the three growth curve component scores (*initial, linear, and quadratic*) for each of the 2 persons

Growth curve model
time coding 0, 1, 2

Exercise 1: solution for person 1
time coding 0, 1, 2

Person 1 has latent state scores 6, 9, and 17. Therefore we have to solve the 3 equations

$$6 = initial$$

$$9 = initial + linear + quadratic$$

$$17 = initial + 2 \cdot linear + 4 \cdot quadratic$$

For the unknown growth curve component scores *initial, linear* and *quadratic* of person 1.

Growth curve model
time coding 0, 1, 2

Since *initial* = 6 we just have to solve the 2 equations

$9 = 6 + linear + quadratic$

$17 = 6 + 2 \cdot linear + 4 \cdot quadratic.$

Substracting 2 times the first equation from the second one,

$-1 = -6 + 2 \cdot quadratic$,

yields: *quadratic* = 2.5. Inserting this result in the first equation,

$9 = 6 + linear + 2.5 = 8.5 + linear$

results in: *linear* = 0.5.

Growth curve model
time coding 0, 1, 2

Solution to Exercise 1 (continued for person 2)
time coding 0, 1, 2

Person 2 has latent state scores 8, 7, and 2. Therefore we have to solve the 3 equations

$8 = initial$

$7 = initial + linear + quadratic$

$2 = initial + 2 \cdot linear + 4 \cdot quadratic$

For the unknown growth curve component scores *initial, linear* and *quadratic* of person 2.

Growth curve model
time coding 0, 1, 2

Since *initial* = 8 we just have to solve the 2 equations

$$7 = 8 + linear + quadratic$$

$$2 = 8 + 2 \cdot linear + 4 \cdot quadratic.$$

Substracting 2 times the first equation from the second one,

$$-12 = -8 + 2 \cdot quadratic\ ,$$

yields: $quadratic = -2$. Inserting this result in the first equation,

$$7 = 8 + linear - 2 = 6 + linear$$

results in: $linear = 1$.

Growth curve model
time coding 0, 1, 2

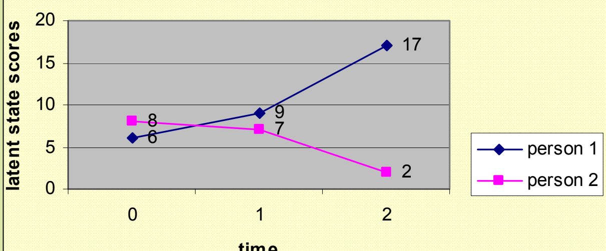
Latent state trajectories of
2 persons - time coding 0, 1, 2

$\eta_1 = 1 \cdot initial$

$\eta_2 = 1 \cdot initial + 1 \cdot linear + 1 \cdot quadratic$

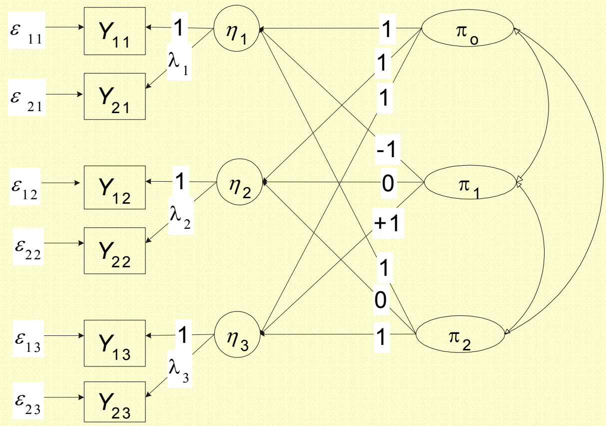
$\eta_3 = 1 \cdot initial + 2 \cdot linear + 4 \cdot quadratic$

person	initial	linear	quadratic
1	6	0.5	2.5
2	8	1	–2



Growth curve model
time coding -1, 0, 1

Choosing time coding t_1 = -1, t_2 = 0, and t_3 = +1, this yields:



with *level* := π_0, *linear* := π_1, and *quadratic* := π_2.

Exercise 2 - time coding –1, 0, 1

The latent state scores η_1, η_2 and η_3 can be translated into growth curve component scores by solving the following 3 equations:

$$\eta_1 = 1 \cdot level + (-1) \cdot linear + 1 \cdot quadratic$$

$$\eta_2 = 1 \cdot level + 0 \cdot linear + 0 \cdot quadratic$$

$$\eta_3 = 1 \cdot level + 1 \cdot linear + 1 \cdot quadratic\ .$$

Compute the three growth curve component scores (*level, linear, and quadratic*) for each of the 2 persons.

Exercise 2: solution for person 1
time coding –1, 0, 1

Person 1 has latent state scores 6, 9, and 17. Therfore we have to solve the 3 equations

$$6 = level - linear + quadratic$$
$$9 = level$$
$$17 = level + linear + quadratic$$

For the unknown growth curve component scores *level, linear* and *quadratic* of person 1.

Since *level* = 9 we just have to solve the 2 equations

$$6 = 9 - linear + quadratic$$
$$17 = 9 + linear + quadratic.$$

Adding the two equations,

$$23 = 18 + 2 \cdot quadratic\ ,$$

yields: *quadratic* = 2.5. Inserting this result in the second equation,

$$17 = 9 + linear + 2.5 = 11.5 + linear$$

results in: *linear* = 5.5.

Exercise 2: solution for person 2
time coding –1, 0, 1

Person 2 has latent state scores 8, 7, and 2. Therfore we have to solve the 3 equations

$$8 = \mathit{level} - \mathit{linear} + \mathit{quadratic}$$
$$7 = \mathit{level}$$
$$2 = \mathit{level} + \mathit{linear} + \mathit{quadratic}$$

For the unknown growth curve component scores *level, linear* and *quadratic* of person 2.

Since $\mathit{level} = 7$ we just have to solve the 2 equations

$$8 = 7 - \mathit{linear} + \mathit{quadratic}$$
$$2 = 7 + \mathit{linear} + \mathit{quadratic}.$$

Adding the two equations,

$$10 = 14 + 2 \cdot \mathit{quadratic} \, ,$$

yields: $\mathit{quadratic} = -2$. Inserting this result in the second equation,

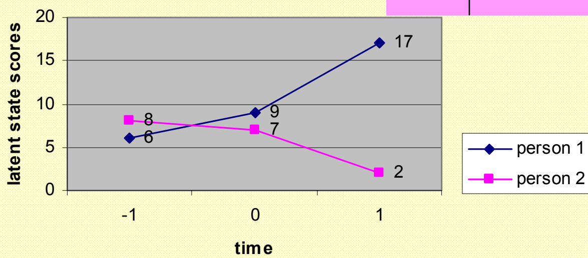
$$2 = 7 + \mathit{linear} - 2 = 5 + \mathit{linear}$$

results in: $\mathit{linear} = -3$.

Latent state trajectories of 2 persons
time coding –1, 0, 1

$$\eta_1 = 1 \cdot level + (-1) \cdot linear + 1 \cdot quadratic$$
$$\eta_2 = 1 \cdot level + 0 \cdot linear + 0 \cdot quadratic$$
$$\eta_3 = 1 \cdot level + 1 \cdot linear + 1 \cdot quadratic\ .$$

person	level	linear	quadratic
1	9	5.5	2.5
2	7	–3	–2



Exercises 1 and 2: Conclusions for time coding

For time coding 0, 1, 2 the constant growth component π_0 is the initial status or the latent state at time = 0, the *first* time point. For time coding –1, 0, 1 the constant growth component π_0 is also the level or the latent state at time 0, but for this coding time 0 is the *second* time point.

For time coding –1, 0, 1 the individual linear growth curve components are the averages of the change between time 2 minus 1 and the change between time 3 minus 2. For person 1 the average change is $[(9 - 6)] + (17 - 9)] / 2 = 5.5$ and for person 2 it is $[(7 - 8)] + (2 - 7)] / 2 = -3$.) The quadratic growth curves are the same for the two time codings.