Latent State-Trait Models

Based on:

Steyer, R., Ferring, D., und Schmitt, M. J. (1992). States and Traits in Psychological Assessment. *European Journal of Psychological Assessment*, *8*, 79-98.

Steyer, R., Schmitt, M. & Eid, M. (1999). Latent state-trait theory and research in per-sonality and individual differences. *European Journal of Personality*, *13*, 389–408.

Eid, M. (2000). A multitrait-multimethod model with minimal assumptions. *Psychometrika*, *65*, 241-261.

Eid, M., Lischetzke, T., Trierweiler, L. I. & Nußbeck, F. W. (2003). Separa-ting trait effects from trait-specific method effects in multitrait-multimethod models: A multiple indicator CTC(M-1) model. *Psychological Methods*.

Primitives

The set of possible outcomes of the random experiment $\Omega = \Omega_U \times \Omega_{S_1} \times \ldots \times \Omega_{S_t} \times \ldots \times \Omega_{S_n} \times \Omega_{O_1} \times \ldots \times \Omega_{O_t} \times \ldots \times \Omega_{O_n}$

Test-score variables $Y_{it}: \Omega \rightarrow \mathrm{IR}$

Person and situation variables

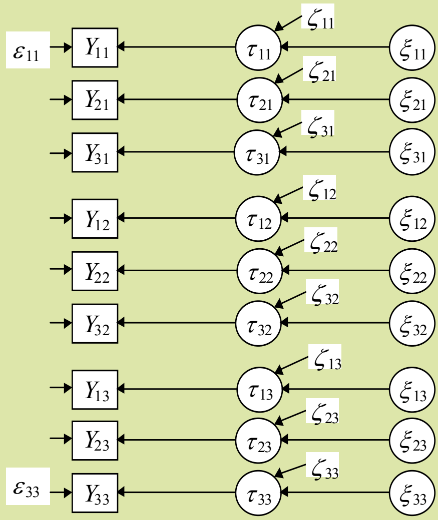
$U: \Omega \rightarrow \Omega_U$ person variable

$S_t: \Omega \rightarrow \Omega_{S_t}$ situation variable

Theoretical Variables

$\tau_{it} := E(Y_{it} \mid U, S_t)$	Latent state variable
$\varepsilon_{it} := Y_{it} - \tau_{it}$	Measurement error variable
$\xi_{it} := E(Y_{it} \mid U)$	Latent trait variable
$\zeta_{it} := \tau_{it} - \xi_{it}$	Latent state residual

Path Diagram



Properties of Latent Variables

Decomposition of variables

$Y_{it} = \tau_{it} + \varepsilon_{it}$

$\tau_{it} = \xi_{it} + \zeta_{it}$

Decomposition of variances

$Var(Y_{it}) = Var(\tau_{it}) + Var(\varepsilon_{it})$

$Var(\tau_{it}) = Var(\xi_{it}) + Var(\zeta_{it})$

Expected Values and Covariances of the Residuals

$$E(\varepsilon_{it}) = 0$$

$$E(\zeta_{it}) = 0$$

$$Cov(\varepsilon_{it},\ \zeta_{js}) = 0$$

$$Cov(\varepsilon_{it},\ \tau_{js}) = 0$$

$$Cov(\varepsilon_{it},\ \xi_{js}) = 0$$

$$Cov(\zeta_{it},\ \xi_{js}) = 0$$

Expected Values: A Little Proof

Proof of: $E(\zeta_{it}) = 0$

$E(\zeta_{it}) = E(\tau_{it} - \xi_{it})$ (Inserting the definition)

$= E(\tau_{it}) - E(\xi_{it})$ (Rule iii of Rule Box 5.1)

$= E[E(Y_{it} \mid U, S_t)] - E[E(Y_{it} \mid U)]$ (Inserting the definitions)

$= E(Y_{it}) - E(Y_{it}) = 0$ (Rule iv of Rule Box 6.2)

Covariances of the Residuals: A Little Proof

Proof of: $Cov(\zeta_{it}, \xi_{js}) = 0$

First we proof $E(\zeta_{it} \mid \xi_{js}) = 0$, which implies $Cov(\zeta_{it}, \xi_{js}) = 0$.

$\zeta_{it} = \tau_{it} - \xi_{it}$ (Definition of ζ_{it})

$= \tau_{it} - E(Y_{it} \mid U)$ (Definition of ξ_{it})

$= \tau_{it} - E[E(Y_{it} \mid U, S_t) \mid U]$ (Rule vi of Rule Box 6.2)

$= \tau_{it} - E(\tau_{it} \mid U)$ (Definition of τ_{it})

This shows that ζ_{it} *is a residual* with respect to the regressor U. Hence, its regression on all functions of U is zero and $\xi_{js} := E(Y_{js} \mid U)$ is such a function of U. Furthermore, its covariances with all functions of U are zero, because the covariances of a residual with all functions of its regressor are zero.

Important Coefficients

Reliability

$$Rel(Y_{it}) = \frac{Var(\tau_{it})}{Var(Y_{it})} = Con(Y_{it}) + Spe(Y_{it})$$

Consistency

$$Con(Y_{it}) = \frac{Var(\xi_{it})}{Var(Y_{it})}$$

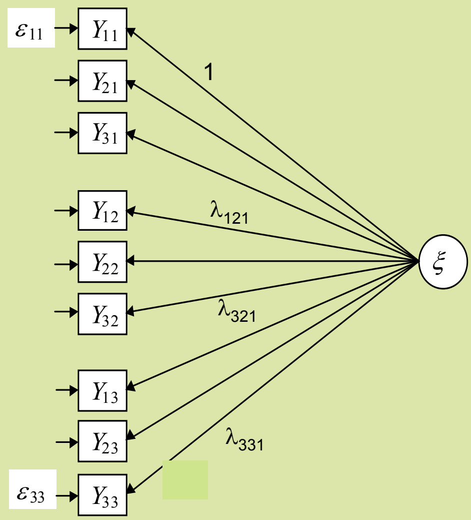
Occasion Specificity

$$Spe(Y_{it}) = \frac{Var(\zeta_{it})}{Var(Y_{it})}$$

Stability of the latent state variable $Kor(\tau_{it}, \tau_{is})$

Stability of the latent trait variable $Kor(\xi_{it}, \xi_{is})$

Singletrait Model
Path Diagram



Singletrait Model
Definition

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$= \lambda_{it0} + \lambda_{it1}\,\xi + \varepsilon_{it}$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0 \qquad (i, t) \neq (j, s)$$

Singletrait Model
Fixing the scale and Identification

Fixing the scale of the latent trait variable ξ:

$$E(\xi) = 0 \quad \text{and} \quad Var(\xi) = 1$$

or $$\lambda_{110} = 0 \quad \text{and} \quad \lambda_{111} = 1$$

Identification in the case $\lambda_{110} = 0$ and $\lambda_{111} = 1$

$$E(\xi) = E(Y_{11})$$

$$\lambda_{it1}^2 Var(\xi) = \frac{Cov(Y_{it}, Y_{js})\ Cov(Y_{it}, Y_{ku})}{Cov(Y_{js}, Y_{ku})}, \quad (i, t) \neq (j, s) \neq (k, u)$$

$$Var(\tau_{it}) = \lambda_{it1}^2 Var(\xi)$$

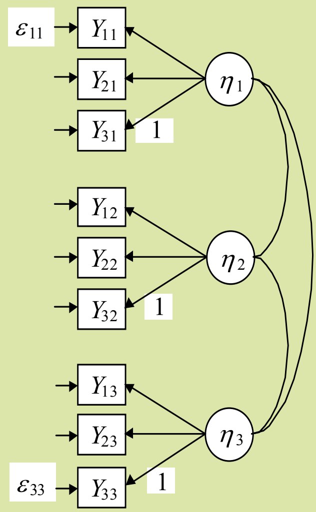
$$Con(Y_{it}) = Rel(Y_{it}) = \frac{\lambda_{it1}^2 Var(\xi)}{Var(Y_{it})}$$

Singletrait Model
Testability

$$\begin{aligned} Var(Y_{it}) &= Var(\tau_{it}) + Var(\varepsilon_{it}) \\ &= \lambda_{it1}^2 Var(\xi) + Var(\varepsilon_{it}) \end{aligned}$$

$$Cov(Y_{it}, Y_{js}) = Cov(\tau_{it}, \tau_{js}) = \lambda_{it1}\lambda_{js1} Var(\xi), \quad (i, t) \neq (j, s)$$

Multistate Model
path diagram



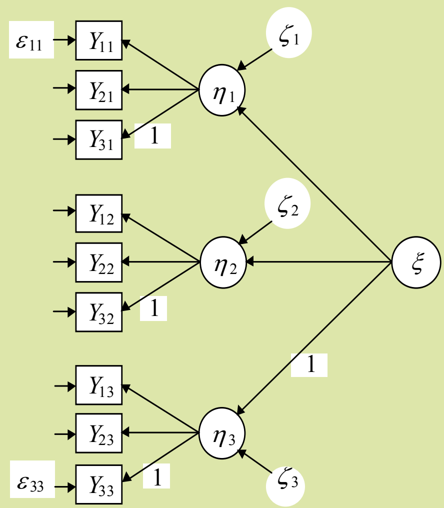
Multistate Model
Definition

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$\tau_{it} = \lambda_{it0} + \lambda_{it1}\,\eta_t + \varepsilon_{it}$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0 \qquad (i, t) \neq (j, s)$$

Multistate-Singletrait Model
path diagram



Multistate-Singletrait Model
Definition

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$= \lambda_{it0} + \lambda_{it1}\eta_t + \varepsilon_{it}$$

$$\eta_t = \gamma_{t0} + \gamma_{t1}\xi + \zeta_t$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0 \qquad (i, t) \neq (j, s)$$

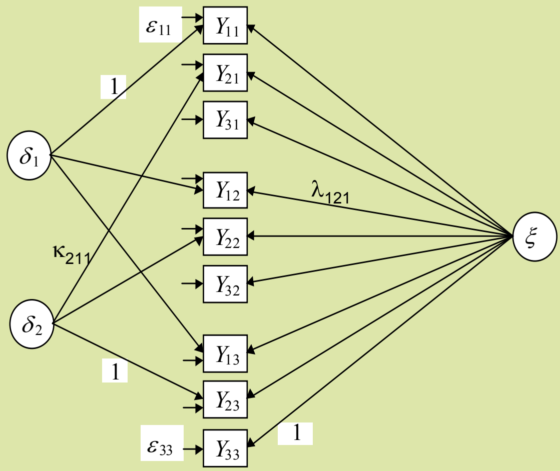
$$Cov(\varepsilon_{it}, \eta_s) = 0$$

$$Cov(\zeta_t, \zeta_s) = 0$$

$$Cov(\zeta_t, \varepsilon_{js}) = 0$$

$$Cov(\zeta_t, \xi) = 0$$

Singletrait Model with Method Factors
path diagram



Singletrait Model with Method Factor
Definition

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$= \lambda_{it0} + \lambda_{it1}\xi + \delta_i + \varepsilon_{it}$$

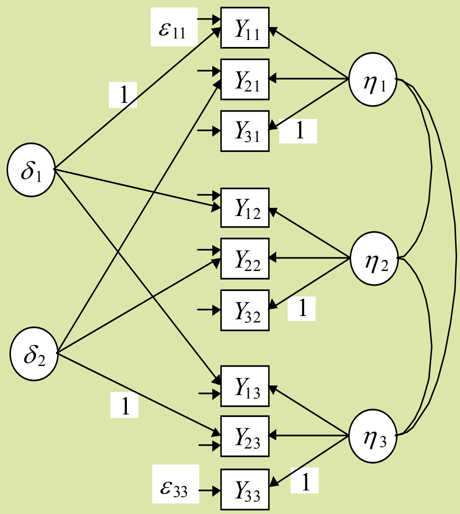
$$Y_{33} = \tau_{33} + \varepsilon_{33}$$

$$= \xi + \varepsilon_{33}$$

$$Cov(\varepsilon_{it}, \varepsilon_{js}) = 0 \qquad (i, t) \neq (j, s)$$

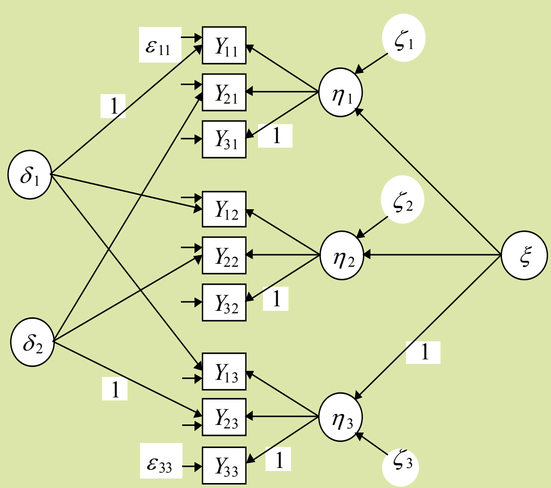
Multistate Model with Method Factors

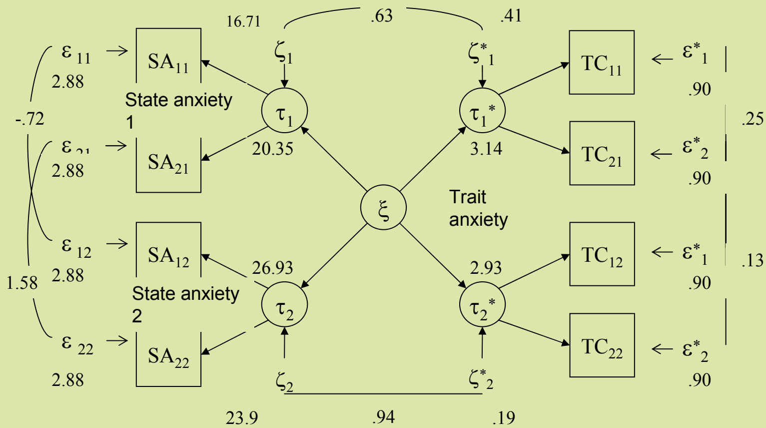
path diagram

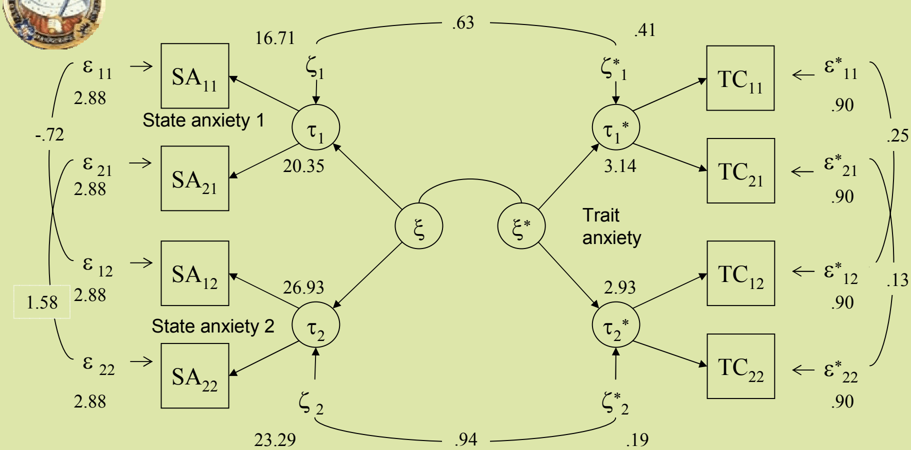


Singletrait-Multistate Model with Method Factors

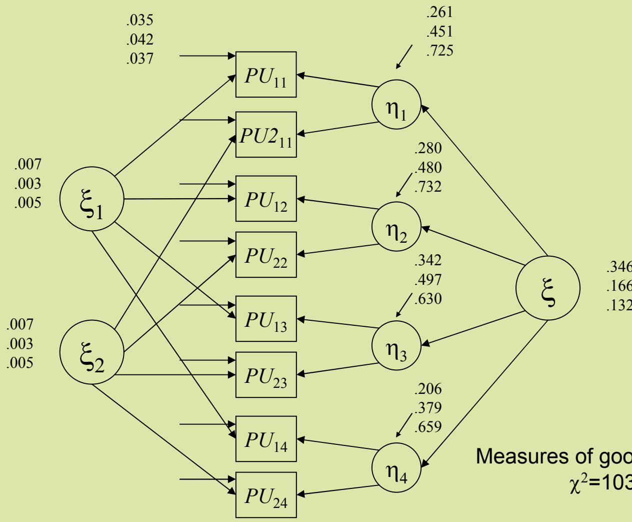
path diagram







χ^2=46.65, *P*=0.00; goodness-of-fit (adj.) = 0.91
absolutely largest standardized residual = 4.03



Measures of goodness-of-fit:
χ^2=103.39, *P*=0.11