Lecture
„Item Response Theory“

Graded Response Models

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Graded Response Models (GRM)

Again we consider items Y_i, $i = 1, \ldots, m$, with ordered response categories (values) $y = 0, 1, \ldots, k_i$ (e.g., Likert scales, ...)

Assumption 1:

- In the logistic Graded Response Model, we assume (the model equation)

$$P(Y_i \geq y \mid U) = P(Y_i \geq y \mid \xi) = \frac{\exp\left[\alpha_i(\xi - \beta_{iy})\right]}{1 + \exp\left[\alpha_i(\xi - \beta_{iy})\right]}, \quad \alpha_i, \beta_{iy} \in \mathbb{R},$$

- In the Probit Graded Response Model we assume

$$P(Y_i \geq y \mid U) = P(Y_i \geq y \mid \xi) = \Phi\left[\alpha_i(\xi - \beta_{iy})\right], \quad \alpha_i, \beta_{iy} \in \mathbb{R},$$

  where Φ denotes the distribution function of the standard normal distribution.

These equations are analog to a 2PL-Birnbaum model, where we replace the event $\{Y = 1\}$ by the event $\{Y \geq y\}$.

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Assumption 2: U-conditional independence of the items $Y_1, \ldots, Y_m$

$$\forall\, i \in \{1, \ldots, m\},\ \ \forall\, y \in \{1, \ldots, k_i\}:$$
$$P(Y_i = y \mid U, Y_1, \ldots, Y_{i-1}, Y_{i+1}, \ldots, Y_m) \;=\; P(Y_i = y \mid U).$$

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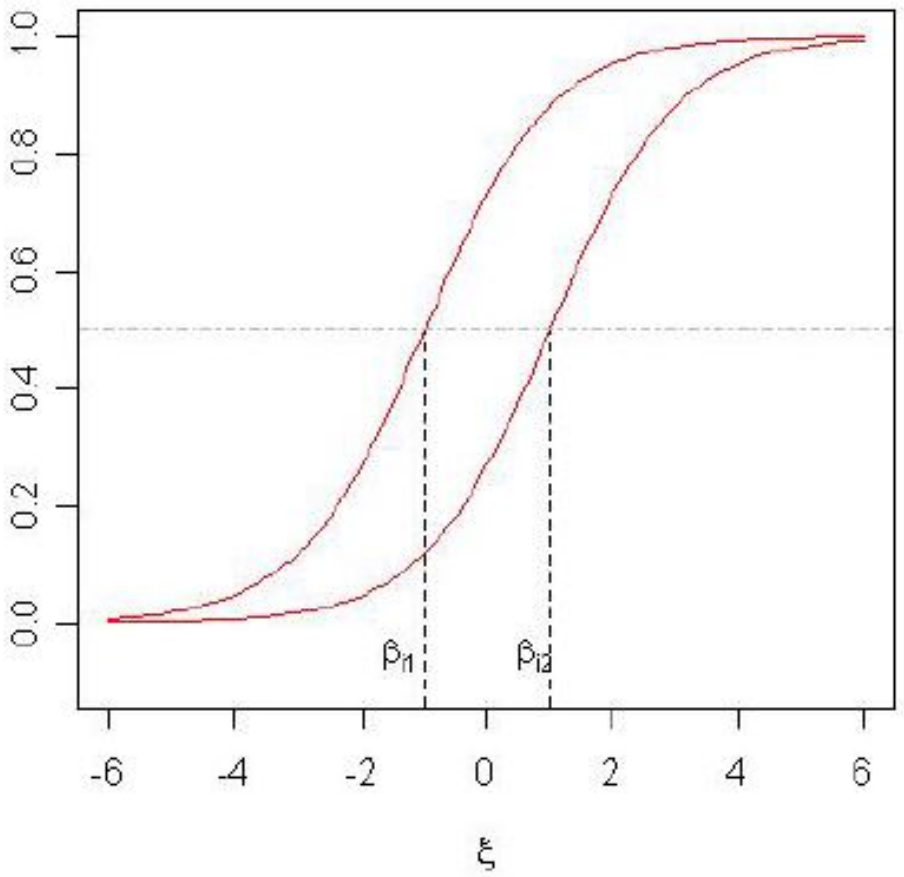

The conditional probabilities $P(Y_i \geq y \mid \xi)$ are also called Operation Characteristic Curves (OCC). These curves are represented for a 3-categorical item Y_i with parameters

$$\alpha_i = 1$$
$$\beta_{i1} = -1$$
$$\beta_{i2} = 1$$

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Category probabilities

- The category probabilities $P(Y_i = y \mid \xi)$ can be computed as the difference between two neighbored OCC. That is, for an item Y_i with k_i categories,

$$
\begin{aligned}
P(Y_i = 0 \mid \xi) &= 1 - P(Y_i \geq 1 \mid \xi) \\
P(Y_i = 1 \mid \xi) &= P(Y_i \geq 1 \mid \xi) - P(Y_i \geq 2 \mid \xi) \\
P(Y_i = 2 \mid \xi) &= P(Y_i \geq 2 \mid \xi) - P(Y_i \geq 3 \mid \xi) \\
&\vdots \\
P(Y_i = k_i \mid \xi) &= P(Y_i \geq k_i \mid \xi).
\end{aligned}
$$

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- **Parameters in GR models**

1. **Threshold parameters**
   - For $i = 1, \ldots, m$ and $y = 1, \ldots, k_i$:

$$P(Y_i \geq y \mid \xi = \beta_{iy}) = 0.5$$

The conditional probability $P(Y_i = y \mid \xi)$ has its maximum at

$$\xi = \frac{\beta_{iy} + \beta_{i,y+1}}{2}.$$

2. **Itemdiskrimination**
   - The discrimination parameter α_i determines the slope of the *OCC*

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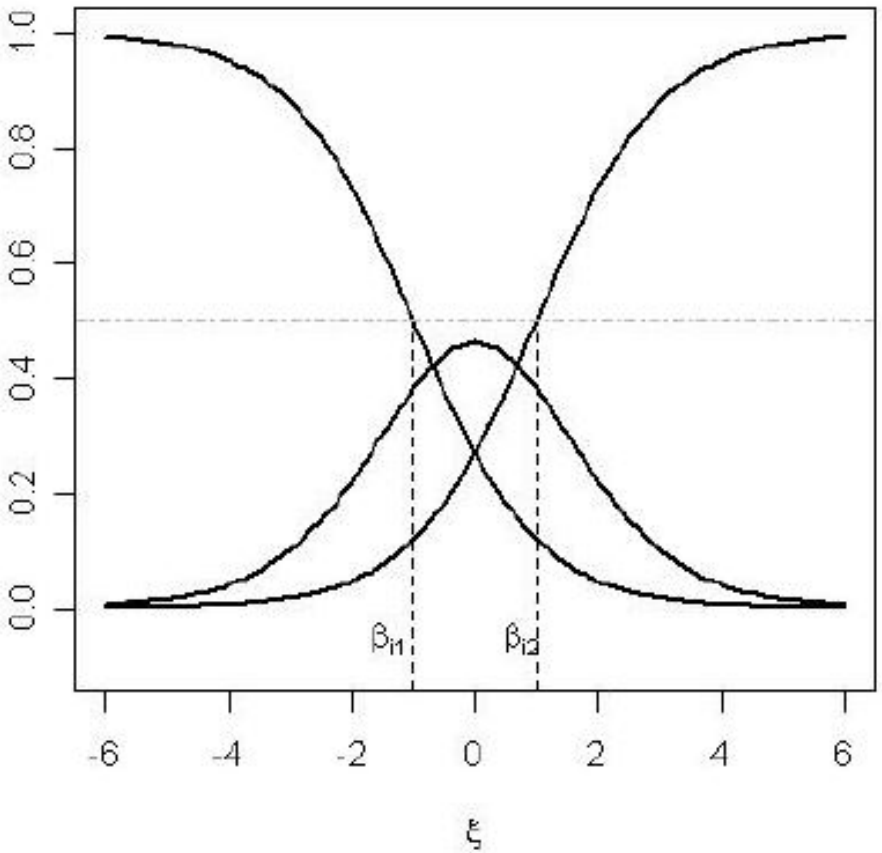

ξ-conditional category probabilities for a 3-categorical item Y_i

$$\alpha_i = 1$$
$$\beta_{i1} = -1$$
$$\beta_{i2} = 1$$

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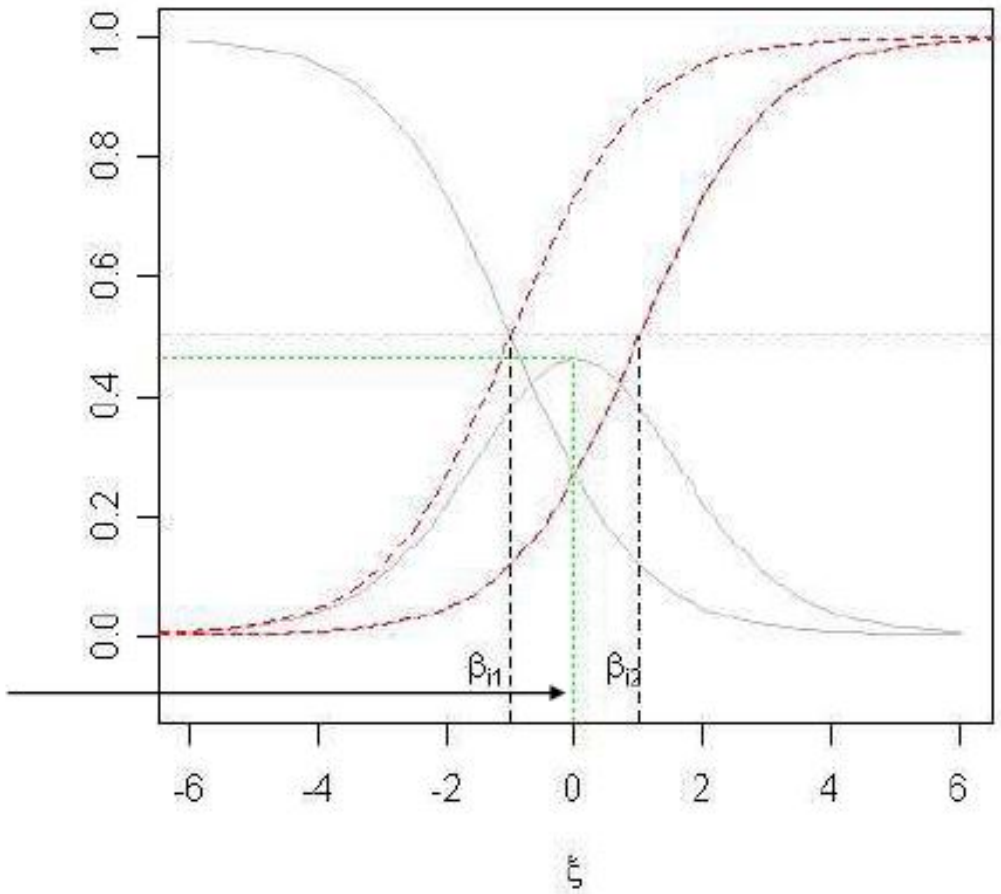
Die Schwellenparameter bestimmen ...

... die Lokation der *OCC*

... die Lokation der Funktionen der Kategorienwahrscheinlich-keiten

$$\frac{\beta_{i1} + \beta_{i2}}{2}$$



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- In the probit GR, the Model parameters can be computed from the factor loadings and the threshold parameters of the factor analytic representation of the probit model.