Model of essentially τ-equivalent tests

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Model of essentially τ-equivalent tests - Assumptions

Definition: Assumptions (a_2) and (b)

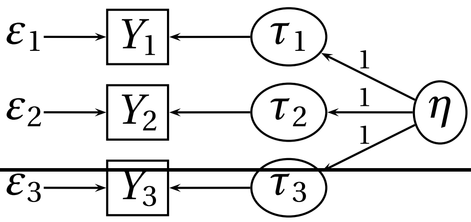
- (a_2) essentially τ-equivalence: $\tau_i = \tau_j + \lambda_{ij}, \quad \lambda_{ij} \in \mathbb{R}$
- (b) uncorrelated errors: $Cov(\varepsilon_i, \varepsilon_j) = 0, \; i \neq j$

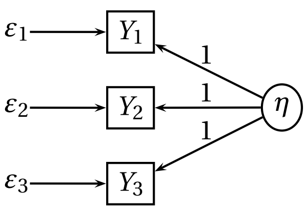
Implication:
(a_2) implies the existence of a latent variable, say η, with:

$\tau_i = \eta - \lambda_i$ (1)
and
$Y_i = \eta - \lambda_i + \varepsilon_i$ (2)
for all $i = 1, \ldots, m$

Proof:
With $\eta := \tau_1$ and $\lambda_i := -\lambda_{i1}$, equations (1) and (2) follow from (a_2).

Path diagram





Implied covariance structure

$$\begin{aligned}
Cov(Y_1, Y_2) &= Cov(\eta - \lambda_1 + \varepsilon_1, \eta - \lambda_2 + \varepsilon_2) \\
&= Cov(\eta, \eta) + Cov(\eta, \varepsilon_2) + Cov(\varepsilon_1, \eta) + Cov(\varepsilon_1, \varepsilon_2) \\
&= Var(\eta) \\
&= \sigma_\eta^2
\end{aligned}$$

$$\begin{aligned}
Var(Y_i) &= Var(\eta) + Var(\varepsilon_i) \\
&= \sigma_\eta^2 + \sigma_{\varepsilon_i}^2
\end{aligned}$$

Implied covariance matrix

Implied covariance matrix for 3 essentially τ-equivalent tests:

$$\begin{bmatrix}
\sigma_\eta^2 + \sigma_{\varepsilon_1}^2 & \sigma_\eta^2 & \sigma_\eta^2 \\
\sigma_\eta^2 & \sigma_\eta^2 + \sigma_{\varepsilon_2}^2 & \sigma_\eta^2 \\
\sigma_\eta^2 & \sigma_\eta^2 & \sigma_\eta^2 + \sigma_{\varepsilon_3}^2
\end{bmatrix}$$

Testability

In the total population, the model implies:

$$Cov(Y_i, Y_j) = \sigma_\eta^2, \quad i \neq j$$

In each subpopulation s, the model implies:

$$\begin{aligned} Cov^{(s)}(Y_i, Y_j) &= \sigma_\eta^{(s)2}, \quad i \neq j \\ E^{(s)}(Y_i) - E^{(s)}(Y_j) &= E^{(s)}(Y_i - Y_j) \\ &= \lambda_{ij} \end{aligned}$$

The constant λ_{ij} is the same number in each subpopulation.

Fixing the scale and identification

Fixing the scale of η : either by $\eta := \tau_1$ or by $E(\eta) = 0$, for example.

Identification:

$$\begin{aligned} Var(\eta) &= Cov(Y_i, Y_j), \quad i \neq j \\ Var(\varepsilon_i) &= Var(Y_i) - Cov(Y_i, Y_j), \quad i \neq j \\ Rel(Y_i) &= \frac{Cov(Y_i, Y_j)}{Var(Y_i)}, \quad i \neq j \end{aligned}$$

Uniqueness of the theoretical concepts

The latent variable η and the coefficients λ_i are uniquely defined (by the model) up to translations.

That is, η and λ_i have a *difference scale*.

Meaningfulness

The following propositions have invariant truth values under translations of η and λ_i

- $\eta(\omega_1) - \eta(\omega_2), \quad$ for all $\omega_1, \omega_2 \in \Omega$
- $\lambda_i - \lambda_j$
- $Var(\eta)$
- $Rel(Y_i)$