Lecture
„Item Response Theory“

Repeating the Essentials of Classical Test Theory

20. Oktober 2016

Organizational Issues

Modul MPSYA101 in the Master Curriculum of Psychology:

- 5 Credit points
- Conditions for earning the credit points
  - Participating at the lecture
  - Participating at the Exercise Course
  - Mastering the written examen

Klausur (90 minutes)

- Multiple choice questions

Readings

1. Embretson, S. E. & Reise, S. P. (2000). *Item Response Theory for Psychologists*. London: Lawrence Erlbaum Associates. Wilson, M. (2004). [Kap. 4-9]
2. Rost, J. (1996). *Lehrbuch Testtheorie Testkonstruktion*. Bern: Huber. [Kap. 3.1 - 3.3, Kap. 4 und 5]
3. Steyer, R. & Eid, M. (2001). *Messen und Testen*. Berlin: Springer. [Kap. 1,9,10,11,13,14,16-17 (18)]
4. Steyer, R., Mayer, A., Geiser, C. & Cole, D. (2015) A Theory of States and Traits - Revised, *Annual Review of Clinical Psychology*, *11*, 71-98.

Repetition - Lecture: „Theories of Psychometric Tests I"

Contents:

- Concepts of Classical Test Theory
  - True-score variable
  - Measurement error
- Models of Classical Test Theory
  - Model of parallel tests
  - Model of essential τ-equivalent tests
  - Model of τ-congeneric tests

The random experiment considered

- Models of Classical Test Theory (and models of IRT) are stochastic measurement models
- The test score variables $Y_1, ..., Y_i, ..., Y_m$ are **random variables**, that are defined referring to a random experiment and its formal representation, a probability space $(\Omega, \mathscr{A}, P)$.
- **The kind of random experiment considered**
    - draw a person u out of the set Ω_U of persons
    - register the values $y_1, ..., y_i, ..., y_m$

Basic concepts of Classical Test Theory (CTT)

Starting point:

Set of possible outcomes $\Omega = \Omega_U \times \Omega_O$

Test-score variables $Y_i : \Omega \rightarrow \mathbb{R}$

Person variable $U : \Omega \rightarrow \Omega_U$

Definition of the theoretical variables:

True-score variables $\tau_i := E(Y_i \mid U)$

Measurement error variables $\varepsilon_i := Y_i - E(Y_i \mid U)$

Basic concepts of Classical Test Theory (CTT)

Properties of true-score and measurement error variables that are implied by their definition:

Decomposition of the variables

$$Y_i = \tau_i + \varepsilon_i \tag{1}$$

Decomposition of the variances

$$Var(Y_i) = Var(\tau_i) + Var(\varepsilon_i) \tag{2}$$

Basic concepts of Classical Test Theory (CTT)

Other properties of the true-score and the measurement error variables, which follow from their definition:

$$Cov(\tau_i, \varepsilon_j) = 0 \qquad (3)$$

$$E(\varepsilon_i) = 0 \qquad (4)$$

$$E(\varepsilon_i \mid U) = 0. \qquad (5)$$

For each mapping $f(U)$ of U:

$$E[\varepsilon_i \mid f(U)] = 0. \qquad (6)$$

Basic concepts of Classical Test Theory (CTT)

Important parameters

Reliability: (Global) parameter for the dependability of Y_i

$$Rel(Y_i) := \frac{Var(\tau_i)}{Var(Y_i)}$$

Error variance: (Global) parameter quantifying the unreliability of Y_i

$$Var(\varepsilon_i)$$

Conditional error variance: Parameter quantifying the conditional unreliability of Y_i

$$Var(\varepsilon_i \mid U = u)$$

Assumptions of Classical Test Theory

(a_1)	τ-equivalence	$\tau_i = \tau_j$
(a_2)	essential τ-equivalence	$\tau_i = \tau_j + \lambda_{ij}$, $\lambda_{ij} \in \mathbb{R}$
(a_3)	τ-congenerity	$\tau_i = \lambda_{ij0} + \lambda_{ij1}\tau_j$, $\lambda_{ij0}, \lambda_{ij1} \in \mathbb{R}, \quad \lambda_{ij1} > 0$
(b)	uncorrelated errors	$Cov(\varepsilon_i, \varepsilon_j) = 0, \quad i \neq j$
(c)	equal error variances	$Var(\varepsilon_i) = Var(\varepsilon_j)$

Models of Classical Test Theory

Models that are defined using some of the following assumptions:

- Parallel tests: (a_1), (b) and (c)
- Essential τ-equivalent tests: (a_2) and (b)
- τ-congeneric tests: (a_3) and (b)