



Vorlesung „Item Response Theory“

The 3PL Birnbaum model

28. November 2016



3PL-Birnbaum model

- Model equation

$$P(Y_i = 1 | \xi) = \gamma_i + (1 - \gamma_i) \frac{\exp[\alpha_i \cdot (\xi - \beta_i)]}{1 + \exp[\alpha_i \cdot (\xi - \beta_i)]}, \quad \forall i = 1, \dots, m.$$

- Meaning of the model parameters:

- 1 Person parameter is a value of the latent variable ξ (e.g., ability, ...)

- 2 Item parameter:

β_i : Item difficulty (same interpretation as in the Rasch model)

α_i : Item discrimination

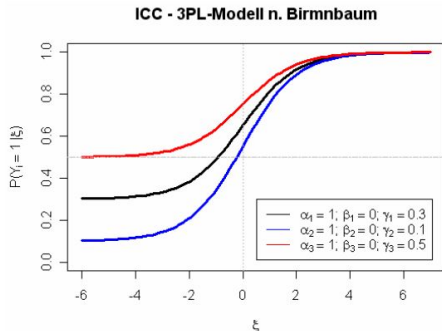
γ_i : Guessing parameter

- note that $\gamma_i \in [0, 1]$



ICC in the 3PL-Birnbaum model

- The meaning of the difficulty parameter depends on the guessing parameter



If $\gamma_i > 0$, then the item difficulty is not the value x of ξ , with $P(Y_i = 1 | \xi = x) = .5$.



3PL-Birnbaum model

- Conditional variance given ξ :

$$\text{Var}(Y_i | \xi) = P(Y_i = 1 | \xi) \cdot P(Y_i = 0 | \xi)$$

- Item information:

$$I(Y_i | \xi) = \alpha_i^2 \cdot \frac{P(Y_i = 0 | \xi)}{P(Y_i = 1 | \xi)} \cdot \left[\frac{P(Y_i = 1 | \xi) - \gamma_i}{1 - \gamma_i} \right]^2$$

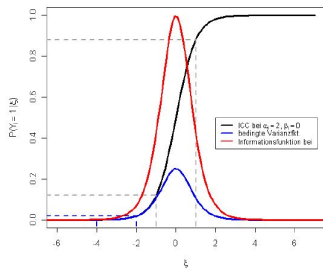
- Test information for m dichotomous items:

$$\begin{aligned} T(\xi) &= \sum_{i=1}^m I(Y_i | \xi) \\ &= \sum_{i=1}^m \alpha_i^2 \cdot \frac{P(Y_i = 0 | \xi)}{P(Y_i = 1 | \xi)} \cdot \left[\frac{P(Y_i = 1 | \xi) - \gamma_i}{1 - \gamma_i} \right]^2 \end{aligned}$$



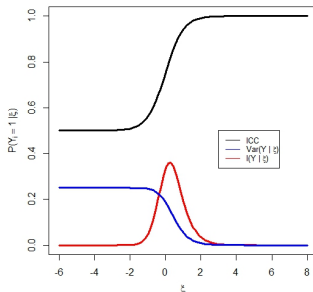
ICC in the 3PL-Birnbaum model

- Conditional variance and information function: 2PL-versus 3PL-model:



$$\alpha_i = 2$$

$$\beta_i = 0$$



$$\alpha_i = 2$$

$$\beta_i = 0$$

$$\gamma_i = 0.5$$