

Methods of Evaluation Research:
Lectures 3 and 4: An outline of the core of the theory

Rolf Steyer

Friedrich Schiller University Jena
Institute of Psychology
Chair of Methodology and Evaluation Research

November 2 and 9, 2015

Introduction	2
Introduction	3
Single-Unit Trials	4
Four Persons	5
Causal Effects Explorer	6
Identification of Causal Effects	7
Core of the Theory	8
Causality Conditions	9
Nonorthogonal ANOVA	10
Nonorthogonal ANOVA	11
Reference	12
References	13

Introduction

In Lecture 2 we treated three causality conditions for the conditional expectation $E(Y|X)$, where we contended ourselves with a dichotomous treatment variable X with values 0 and 1:

- $E(Y|X) = E^{\bar{U}}(Y|X)$ causal unbiasedness of $E(Y|X)$, which is equivalent to: For $x = 0, 1$: $E(Y|X=x) = E(\tau_x)$, where $\tau_x := E^{X=x}(Y|U)$.
- $P(X=1|U) = P(X=1)$ independence of X and U
- $E(Y|X, U) = E(Y|X)$ X -conditional mean independence of Y from U

The last two conditions imply the first, and they can be tested empirically.

However, these causality conditions require more than we actually need for causal inference, *provided that there are covariates*.

If we consider only the total effect of a treatment variable, then a *covariate* is any pre-treatment variable such as gender, educational status, a pretest (measured before the onset of the treatment), etc. Note that gender and educational status are (deterministic) functions of U , whereas a pretest (e.g., depression before treatment) is not, because it can be measured only with measurement error.

Single-Unit Trial

Which type of *empirical phenomenon* do we consider from now on?

- Drawing a person u out of a set of persons. This value u is a value of the random variable U .
- observing the value z of (a possibly multivariate qualitative or quantitative and possibly fallible) covariate Z_{all} of the unit
- assigning the unit or observing its assignment to one of several experimental conditions (represented by the value x of the treatment variable X),
- recording the numerical value y of the outcome variable Y .

Example with four persons

Table 1: Four Persons With Self-Selection to Treatment

Unit Treatment Success	Outcomes ω $P(\{\omega\})$	Observables				Conditional expectations			
		Person variable U	Sex Z	Treatment variable X	Outcome variable Y	$E(Y X, U) =$ $P(Y=1 X, U)$	$E(Y X, Z) =$ $P(Y=1 X, Z)$	$E(Y X) =$ $P(Y=1 X)$	$P(X=1 U)$
(Joe, no, -)	.0675	Joe	m	0	0	.7	.66	.63	.1
(Joe, no, +)	.1575	Joe	m	0	1	.7	.66	.63	.1
(Joe, yes, -)	.0050	Joe	m	1	0	.8	.44	.44	.1
(Joe, yes, +)	.0200	Joe	m	1	1	.8	.44	.44	.1
(Jim, no, -)	.0175	Jim	m	0	0	.3	.66	.63	.9
(Jim, no, +)	.0075	Jim	m	0	1	.3	.66	.63	.9
(Jim, yes, -)	.1350	Jim	m	1	0	.4	.44	.44	.9
(Jim, yes, +)	.0900	Jim	m	1	1	.4	.44	.44	.9
(Sue, no, -)	.0600	Sue	f	0	0	.7	.60	.63	.2
(Sue, no, +)	.1400	Sue	f	0	1	.7	.60	.63	.2
(Sue, yes, -)	.0200	Sue	f	1	0	.6	.44	.44	.2
(Sue, yes, +)	.0300	Sue	f	1	1	.6	.44	.44	.2
(Ann, no, -)	.0400	Ann	f	0	0	.2	.60	.63	.8
(Ann, no, +)	.0100	Ann	f	0	1	.2	.60	.63	.8
(Ann, yes, -)	.1200	Ann	f	1	0	.4	.44	.44	.8
(Ann, yes, +)	.0800	Ann	f	1	1	.4	.44	.44	.8

www.metheval.uni-jena.de

5 / 13

Causal Effects Explorer

www.causal-effects.de

Kassel0.tab: self-selection and biased conditional expectations $E(Y|X)$ and $E(Y|X, Z)$ Kassel1.tab: Z -conditional independence between X and U Kassel2.tab: Z -conditional mean independence of Y from U In the last two examples, the conditional expectations $E(Y|X, Z)$ are unbiased, where Z denotes sex.In other words, the conditional expectations $E^{Z=1}(Y|X)$ (for males) and $E^{Z=2}(Y|X)$ (for females) are unbiased.

How do you compute the average total effect in this case?

www.metheval.uni-jena.de

6 / 13

Unbiasedness of the conditional expectation $E(Y|X, Z)$ **Notation**

X — treatment variable Z — covariate, i.e., $Z = f(U, Z_{all})$

Y — outcome variable U — person variable

Z_{all} — vector of all observed covariates

$\tau_x := E^{X=x}(Y|U, Z_{all})$, $x = 0, 1$, — true outcome variables

$\delta_{10} := \tau_1 - \tau_0$ — atomic total-effect variable

$E(\delta_{10})$ — average total effect

$E(\delta_{10}|Z=z)$ — conditional total effect given $Z=z$

Unbiasedness of the conditional expectation $E^{X=x}(Y|Z)$

$$(1) \quad E^{X=x}(Y|Z) = E(\tau_x|Z)$$

Unbiasedness of $E^{X=x}(Y|Z)$ implies:

$$(2) \quad E[E^{X=x}(Y|Z)] = E[E(\tau_x|Z)] = E(\tau_x)$$

Remember : $E(\delta_{10}) = E(\tau_1) - E(\tau_0)$

and, if $V = f(Z)$,

$$(3) \quad E[E^{X=x}(Y|Z)|V] = E[E(\tau_x|Z)|V] = E(\tau_x|V)$$

Remember : $E(\delta_{10}|V) = E(\tau_1|V) - E(\tau_0|V)$

Other Causality Conditions

Let Z_i , $i = a, \dots, d$, denote (possibly multivariate) covariates, i.e., $Z_i = f(U, Z_{all})$. Then each of the following conditions is sufficient for unbiasedness with $Z = Z_i$:

- Z_a -conditional independence of (U, Z_{all}) and treatments ($X \perp\!\!\!\perp U, Z_{all} | Z_a$)

$$P(X=x|U, Z_{all}) = P(X=x|Z_a), \quad \forall x$$

- Complete cause condition ($Y \vdash U, Z_{all} | X, Z_b$)

$$E(Y|X, U, Z_{all}) = E(Y|X, Z_b)$$

- Z_c -Conditional Strong Causality

if W is (U, Z_{all}) -measurable, then there is a real-valued function h such that
 $E(Y|X, Z_c, W) = E(Y|X, Z_c) + h(Z_c, W)$ and $P(X=x|U, Z_{all}) > 0, \quad \forall x$

- Z_d -conditional independence of true outcomes and treatments
 ($\tau_0, \tau_1 \perp\!\!\!\perp X | Z_d$ "strong ignorability")

$$P(X=x|Z_d, \tau_0, \tau_1) = P(X=x|Z_d) \quad \text{and} \quad P(X=x|U, Z_{all}) > 0, \quad \forall x$$

Note that there are still more causality conditions (see, e.g., ?, ?).

Nonorthogonal ANOVA

Table 2: Expectations in three treatment conditions

treatment	expectation of Y in the treatment conditions $E(Y X=x)$	treatment probabilities $P(X=x)$
$X=0$ (control)	111.25	1/3
$X=1$ (treatment 1)	100.00	1/3
$X=2$ (treatment 2)	114.25	1/3
$E(Y)$	108.50	

Nonorthogonal ANOVA

Table 3: Expectations $E(Y|X=x, Z=z)$ in treatment \times neediness conditions

treat- ment	neediness						$P(X=x)$
	low ($Z=0$)		medium ($Z=1$)		high ($Z=2$)		
$X=0$	120	(20/120)	110	(17/120)	60	(3/120)	(40/120)
$X=1$	100	(7/120)	100	(26/120)	100	(7/120)	(40/120)
$X=2$	80	(3/120)	90	(17/120)	140	(20/120)	(40/120)
$P(Z=z)$	(30/120)		(60/120)		(30/120)		

Note. Probabilities $P(X=x, Z=z)$, $P(Z=z)$, and $P(X=x)$ in parentheses.

Most Recent Publications

Mayer, Thoemmes, Rose, Steyer, and West (2014) Steyer, Mayer, and Fiege (2014)

www.metheval.uni-jena.de

12 / 13

References

Thoemmes, F., Rose, N., Steyer, R., & West, S. G. (2014). Theory and analysis of total, direct, and indirect causal effects. *Multivariate Behavioral Research*, 49(5), 425–442.

Steyer, R., Mayer, A., & Fiege, C. (2014). Causal inference on total, direct, and indirect effects. In Michalos, A. C. (Ed.), *Encyclopedia of Quality of Life and Well-Being Research* (pp. 606–631). Dordrecht, Netherlands: Springer.

www.metheval.uni-jena.de

13 / 13